Accelerated Math III Worksheet
Vectors and Lines

Name______________________
Period____Date_____________

Lines consist of collinear points and so do vectors, so we can think of lines as the terminal points of sums of vectors. In other words, suppose (1, 4) and (5, 7) lie on a line. Well, the vector <4, 3> lies on that line as well - and so does every scalar multiple of the vector <4, 3> - and we could write those d·<4, 3>. But we need to get to that vector, somehow, so we start a vector from the origin (called a position vector) to add to our multiple of <4, 3>. Therefore one vector equation of the line through (1, 4) and (5, 7) is:

$\vec{r} = \langle 1,4 \rangle + d\langle 4,3 \rangle$ or $\vec{r} = \vec{i} + 4\vec{j} + d(4\vec{i} + 3\vec{j})$ or $\vec{r} = 5\vec{i} + 7\vec{j} + d(4\vec{i} + 3\vec{j})$

1. Why are there infinitely many vector equations of a line?

2. Write a vector equation of the line through (3, 1) and (2, 3).

3. Write the equation of the line in problem #2 in slope-intercept form. Then compare and contrast these forms.

4. What is the slope of $\vec{r}$ =<4, 5> + d<-2, 5>? Name 3 points on this line.

5. What are the intercepts of $\vec{r}$ =<3, -4> + d<1, 6>?

6. Write a vector equation of the line through (7, -3) and (–2, 9).

7. Write a vector equation of the line parallel to <6, 2> through (4, –1).

8. Write a vector equation of the line 3x + 7y = 21

9. Write a vector equation of the line y = 3x + 4.

10. Locate the point 80% of the way from (1, 7) to (5, 3). (Use vectors…)

11. A common convention is to write the vector along the line as a unit vector.

A) Write a vector equation of the line through (7, –3) and (–2, 9), if the second vector is a unit vector.

B) Find the angle that the line makes with the positive x-axis. What is the cosine of that angle?

C) Find the angle that the line makes with the positive y-axis. What is the cosine of that angle?

D) When the vector along the line is a unit vector, the x and y coefficients are called direction cosines of the line and are abbreviated c_1 and c_2. The angles made with the x and y axes are called α (alpha) and β (beta). What are the direction cosines and the direction angles for the line: 5x – 12y = 60?

12. Line ℓ contains (2, 7) and (3, 1). Locate all points on ℓ that are 4 units from (2, 7).

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Lines consist of collinear points and so do vectors, so we can think of lines as the terminal points of sums of vectors. In other words, suppose (1, 4) and (5, 7) lie on a line. Well, the vector <4, 3> lies on that line as well - and so does every scalar multiple of the vector <4, 3> - and we could write those d·<4, 3>. But we need to get to that vector, somehow, so we start a vector from the origin (called a position vector) to add to our multiple of <4, 3>. Therefore one vector equation of the line through (1, 4) and (5, 7) is:

$$\vec{r} = \langle 1,4 \rangle + d\langle 4,3 \rangle \quad \text{or} \quad \vec{r} = \vec{i} + 4\vec{j} + d(4\vec{i} + 3\vec{j}) \quad \text{or} \quad \vec{r} = 5\vec{i} + 7\vec{j} + d(4\vec{i} + 3\vec{j})$$

1. Why are there infinitely many vector equations of a line?

For the position vector, you can use any point on the line, and you can use any vector that is a scalar multiple of the original vector representing the slope.

2. Write a vector equation of the line through (3, 1) and (2, 3). **$\vec{r} = \langle 3,1\rangle + d\langle -1,2\rangle$**

3. Write the equation of the line in problem #2 in slope-intercept form. Then compare and contrast these forms. **y = -2x + 7 The vector <-1, 2> gives the Δy and Δx for the slope. The point (3, 1) can be used to find the y intercept. Vector equation is more like point-slope form…**

4. What is the slope of $\vec{r} = \langle 4, 5\rangle + d\langle -2,5\rangle$? Name 3 points on this line. **Slope = 5/-2**

Points include (4, 5), (2, 10), (0, 15), (6, 0)

5. What are the intercepts of $\vec{r} = \langle 3, -4\rangle + d\langle 2, 6\rangle$? **Hint: Solve 3+2d=0… (0, -13) (5/3, 0)**

6. Write a vector equation of the line through (7, -3) and (–2, 9). **$\vec{r} = \langle 7,-3\rangle + d\langle -9,12\rangle$**

7. Write a vector equation of the line parallel to <6, 2> through (4, –1). **$\vec{r} = \langle 4,-1\rangle + d\langle 6,2\rangle$**

8. Write a vector equation of the line 3x + 7y = 21

Examples are: $\vec{r} = \langle 7,0\rangle + d\langle -7,3\rangle$ or $\vec{r} = \langle 0,3\rangle + d\langle 7,-3\rangle$ How can you determine correct answers?

9. Write a vector equation of the line y = 3x + 4.

Examples are: $\vec{r} = \langle 0,4\rangle + d\langle 1,3\rangle$ or $\vec{r} = \langle 1,7\rangle + d\langle 2,6\rangle$ How can you determine correct answers?

10. Locate the point 80% of the way from (1, 7) to (5, 3). (Use vectors…) **(4.2,3.8)**

11. A common convention is to write the vector along the line as a unit vector.

A) Write a vector equation of the line through (7, −3) and (–2, 9), if the second vector is a unit vector. **$\vec{r} = \langle 7,-3\rangle + d\langle -0.6,0.8\rangle$**

B) Find the angle that the line makes with the positive x-axis. **126.870°**
What is the cosine of that angle? **-3/5**

C) Find the angle that the line makes with the positive y-axis. **36.870°**
What is the cosine of that angle? **4/5**

D) When the vector along the line is a unit vector, the x and y coefficients are called direction cosines of the line and are abbreviated c_1 and c_2. The angles made with the x and y axes are called α (alpha) and β (beta). What are the direction cosines and the direction angles for the line: 5x − 12y = 60? **With x axis: Cosine=12/13, Angle=22.620° With y axis: Cosine=-5/13, Angle=112.620°**

12. Line *l* contains (2, 7) and (3, 1). Locate all points on *l* that are 4 units from (2, 7).

(2.6576,3.0544); (1.3424,10.9456)