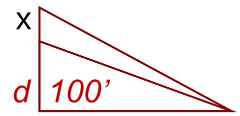
Trig Applications — Name *KEY*
(With Geometry!!) — Period ______ Date ______________

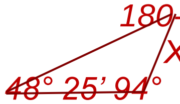
1. A person standing 100' from the bottom of a cliff notices a tower on top of the cliff. The angle of elevation to the top of the cliff is 30° and the angle of elevation to the top of the tower is 58°. How tall is the tower?



tan 30° = d/100 and tan 58° = (d + x)/100
so x = 100tan 58° - 100tan 30° = 102.3'

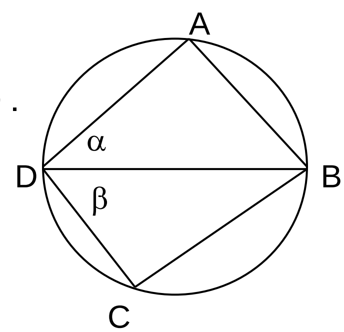
2. Prove that the area of a triangle can be found by the formula: $A = \frac{1}{2}\left(\frac{\sin B \cdot \sin C}{\sin A}\right) \cdot a^2$
A = ½ a·b·sin C and a/sin A = b/sin B, so b = a·sin B/sin A
Substitute for b and get A = ½ a(a·sin B/sin A)·sin C
. *so A = ½(sin B·sin C/sin A)·a^2*

3. A lamppost tilts towards the sun at an angle of 4° from the vertical and casts a 25' shadow when the angle of elevation of the sun is 48°. What is the length of the lamppost?



180-48-94=38 *25/sin 38° = x/sin 48°*
so x = 30.18'

4. Ptolemy's Theorem states that is ABCD is a cyclic quadrilateral (one that is inscribed in a circle), then AB·CD + AD·BC = AC·BD . Consider the special case in which BD is a diameter with length equal to 1.



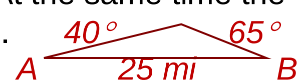
A) State why AB = sin α and that AD = cos α.
ΔABD and ΔCBD are right angles so sin α = AB/1 & cos α = AD/1.

B) Find BC and CD in terms of β.
Sin β = BC/1 and cos β = CD/1

C) Draw AC and, using geometry and the Law of Sines, show, **without** using the formula for the sine of the sum of two angles, that AC = sin(α + β).
By law of sines, AC/sin(α+β) = AD/sin∠DCA = AD/sin∠DBA = AD/cos∠BDA = cosα/cosα.
So AC/sin(α+β) = 1 → AC = sin(α+β)

D) Use parts A), B), C), and Ptolemy's theorem to derive the **formula** for sin(α + β).
AB·CD + AD·BC = AC·BD → sin α· cos β + cos α· sin β = sin(α+β).

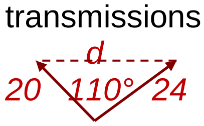
5. Two stargazers are 25 miles apart at locations A and B. One night the stargazer at A looks toward B and sees a UFO at an angle of elevation of 40°. At the same time the stargazer at B looks toward A and sees a UFO at an angle of 65°.



A) How far is the UFO from each of the stargazers?
25/sin 75° = a/sin 40° = b/sin 65° → b = 23.46 mi and a = 16.64 mi

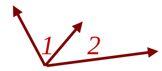
B) How far off the ground is the UFO hovering?
h = 16.64sin65° = 15.08 miles

6. Two snowmobilers start from the same point. One heads N 75° W at a speed of 10 mph and the other heads N 35° E at a speed of 12 mph. After 2 hours they find that their radio transmissions are barely audible. Use this information to determine the range of their radios.



$d^2 = 20^2 + 24^2 - 2(20)(24)\cos 110°$, so d = 36.12 miles

7. Line ***l*** bisects an angle formed by two lines ***l₁*** and ***l₂***. If the slopes of these three lines are m, m_1, and m_2 respectively,



A) Show that: $\dfrac{m_1 - m}{1 + m_1 \cdot m} = \dfrac{m - m_2}{1 + m \cdot m_2}$ *$\tan(\angle 1) = \tan(l_1 - l)$ and $\tan(\angle 2) = \tan(l - l_2)$*

If $\angle 1 \cong \angle 2$, $\tan(\angle 1) = \tan(\angle 2)$. So $= \tan(l_1 - l) = \tan(l - l_2) \rightarrow \dfrac{m_1 - m}{1 + m_1 \times m} = \dfrac{m - m_2}{1 + m \times m_2}$

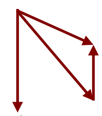
B) If l_1 and l_2 have equations of y = 2x and y = x, find an equation of ***l***.

$$\frac{m_1 - m}{1 + m_1 \times m} = \frac{m - m_2}{1 + m \times m_2} \Rightarrow \frac{2 - m}{1 + 2m} = \frac{m - 1}{1 + m} \Rightarrow 2 + m - m^2 = 2m^2 - m - 1 \Rightarrow m = \frac{1 \pm \sqrt{10}}{3}$$

so equations are $y = \left(\dfrac{1 \pm \sqrt{10}}{3}\right) x$

8. A pilot programs his plane to travel S 36° E at 450 mph and then takes a 3 hour nap. Unfortunately, a 30 mph wind blowing due north affects the path of the plane for the entire trip. When the pilot wakes up, how far from the airport has he actually flown? What is his bearing from the airport?

$d^2 = 1350^2 + 90^2 – 2(1350)(90)\cos 36°$ so d = 1278 miles



1278/sin(36°) = 90/sin k → k = 2.372°, so the bearing is S38.372°E.

9. A parallelogram has two sides of lengths 6 and 9. The included angle between the sides is 68°.

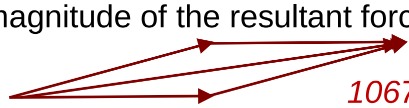
A) How long is the longer diagonal? *$d^2 = 6^2 + 9^2 – 2(6)(9)\cos 68° \rightarrow d = 8.749$*

B) How long is the shorter diagonal? *$d^2 = 6^2 + 9^2 – 2(6)(9)\cos 112° \rightarrow d = 12.55$*

C) What is the angle between the diagonals? *Diagonals bisect each other, so*

$6^2 = 4.374^2 + 6.274^2 – 2(4.374)(6.274)\cos\theta \rightarrow \theta = 65.80°$ or $114.20°$

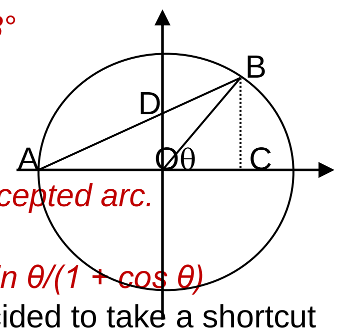
10. Two forces are acting on an object. If the magnitude of one force is 500 N and the magnitude of the second force is 600 N and if the angle between the forces is 28°, find the magnitude of the resultant force and the angle it makes with the 500 N force.



$r^2 = 500^2 + 600^2 – 2(500)(600)\cos 152°$ so r = 1067.6 N

1067.6/ sin152 = 600/sin k → k = 15.298°

11. The diagram to the right shows a unit circle with m∠BOC = θ.



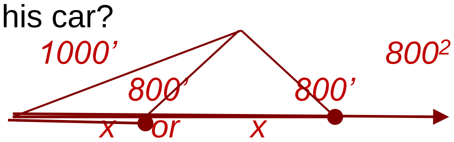
A) Explain why the measure of ∠BAO = θ/2.

The measure of an inscribed angle is half the measure of the intercepted arc.

B) Explain why tan (θ/2) = sin θ/(1 + cos θ).

Look at ΔABC. By SOHCAHTOA, tan A = BC/AC → tan (θ/2) = sin θ/(1 + cos θ)

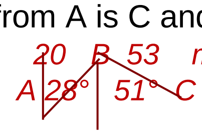
12. Arthur's car broke down along a long straight road and he decided to take a shortcut through the woods. He sets off at an angle of 40° with the road and walks about 1000'. After deciding he is lost, Arthur turns and after walking 800', he finds the road. How far is he from his car?



$800^2 = x^2 + 1000^2 – 2x(1000)\cos 40°$

x = 1242.3 ' or 289.78 '

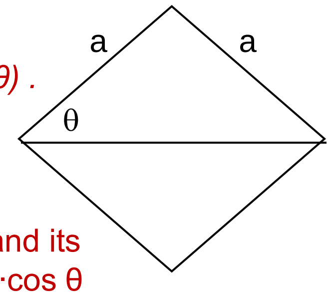
13. Point B is 20 miles N 28° E of point A, and point C is 53 miles at S 51° E of B. How far from A is C and what is the bearing from A to C?



m∠ABC = 28+51=79° $AC^2 = 20^2 + 53^2 – 2(20)(53)\cos 79°$ so AC = 52.957 mi

52.957/sin79 = 53/sin∠BAC → m∠BAC = 79.24°. 79.24+20 = 99.24° so S80.76°E

14. Use the figure to the right.



A) Prove that the area of the rhombus is $a^2 \sin 2\theta$.

Draw other diagonal. In one of those triangles, A = ½ a·a·sin(2θ). Since there are 2 Δs, A = 2(1/2)a²sin(2θ) = a²sin(2θ)

B) Prove (using a property of the diagonals) that the area of the rhombus is $2a^2 \sin\theta\cdot\cos\theta$.

Both diagonals make 4 ≅ right Δs. The base of one is a·cos θ and its height is a·sin θ. So total area is 4(1/2)acos θ·asin θ = 2a²sin θ·cos θ

C) What formula can be deduced from the relationship between A and B?

a²sin(2θ) = 2a²(sin θ)(cos θ) → sin(2θ) = 2sin θ·cos θ

15. The lengths of the sides of a triangle are consecutive integers n, n+1, and n+2. Also, the measure of the largest angle is twice the measure of the smallest angle θ.

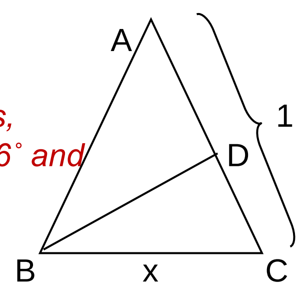
A) Use the Law of Sines to show that $\cos\theta = (n + 2)/(2n)$. n/sin θ = (n+2)/sin(2θ) → *n/sin θ = (n+2)/(2sinθ·cosθ) →n=(n+2)/2cos θ, so cos θ = (n+2)/(2n).*

B) Use the Law of Cosines to show that $\cos\theta = (n + 5)/(2n + 4)$.

$n^2 = (n+1)^2 + (n+2)^2 - 2(n+1)(n+2)\cos\theta \rightarrow n^2 - n^2 - 4n - 4 - n^2 - 2n - 1 = -2(n+1)(n+2)\cos\theta \rightarrow -n^2-6n-5 = -2(n+1)(n+2)\cos\theta \rightarrow -(n+5)(n+1)/-2(n+1)(n+2) = \cos\theta = (n+5)/(2n+4)$

C) Use A) and B) to find n. *(n+2)/(2n) = (n+5)/(2n+4) →(n+2)/(n) = (n+5)/(n+2)→ n= 4*

16. Given that AB = AC = 1, m∠A = 36° and $\overrightarrow{BD}$ bisects ∠B.



A) Prove that ΔABC ~ ΔBCD. *Since ΔABC is isosceles, m∠C = m∠ABC = 72˚. Since $\overrightarrow{BD}$ bisects ∠ABC, m∠CBD = 36˚ and since the triangles share ∠C, by AA~, ΔABC ~ ΔBDC. And since ΔBCD is isosceles, ΔABC ~ ΔBCD.*

B) Use similar triangles to show that $\frac{x}{1-x} = \frac{1}{x}$

Since ΔBCD is isosceles, BD = x. But since m∠ABD = 36˚, ΔABD is also isosceles, so AD = x. Therefore CD = 1 – x. So $\frac{BC}{CD} = \frac{AC}{BC} \Rightarrow \frac{x}{1-x} = \frac{1}{x}$

C) Show that x = $\frac{\sqrt{5}-1}{2}$ *Using the above equation, cross multiply to get x² = 1 - x. Solve x² + x – 1 = 0 using the quadratic formula and get:*

$$x = \frac{-1+\sqrt{1+4}}{2} = \frac{\sqrt{5}-1}{2}$$

D) Draw the bisector of ∠A and show that sin 18° = $\frac{\sqrt{5}-1}{4}$ = cos 72° *Let the foot be called F. Since ∠A was bisected, m∠BAF = 36˚ and since, in an isosceles triangle the angle bisector is also the altitude, m∠AFB = 90˚. Sin 18˚ = opp/hyp = (x/2)/1 =* $\frac{\sqrt{5}-1}{2}\bullet\frac{1}{2} = \frac{\sqrt{5}-1}{4}$ *And by the cofunction property, sin 18˚ = cos 72˚.*

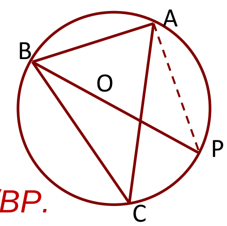
E) Draw the perpendicular from B to AC and show that cos 36° = $\frac{\sqrt{5}+1}{4}$ = sin 54°.

Let the foot be K. ΔBKA is right, so cos 36˚ = AK/AB = (x + ½(1 – x)) = ½ + ½ x = $\frac{1}{2} + \frac{\sqrt{5}-1}{4} = \frac{2}{4} + \frac{\sqrt{5}-1}{4} = \frac{\sqrt{5}+1}{4}$ *= cos 36˚ = sin 54˚.*

F) Explain how to use this information to find the exact value of cos 18° = sin 72°.

We know sin 18˚, so use the fact that cos² 18˚ + sin² 18˚ = 1 and solve for cos 18˚ = sin 72

17. Let circle O be the circumcircle of acute ΔABC and let $\overline{BP}$ be the diameter through B.



A) Show that $\angle BPA \cong \angle BCA$. *Since they are both inscribed angles that intercept arc BA, they must be congruent.*

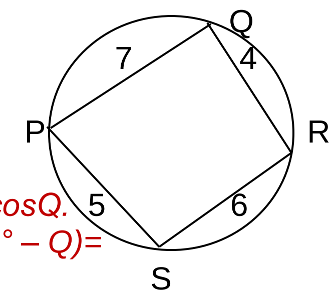
B) Show that AB/BP = sin BPA. *Since $\angle BPA$ is inscribed in a semicircle, it is a right angle so by SOHCAHTOA, sin $\angle BPA$=AB/BP.*

C) Use parts A) and B) to show that c/sin BCA = diameter = a/sin CAB = b/sin ABC. (Name this formula.) *By substitution, AB/BP = sin$\angle$BCA so diameter = AB/sin C$\rightarrow$ diameter = c/sin C. Similarly, $\angle CAB \cong \angle CPB$, so sin$\angle$CPB = sin$\angle$CAB = BC/PB So diameter = a/sin A. Drawing diameter AK could reveal $\angle ABC \cong \angle AKC$ and AK = AC/sin $\angle$AKC = b/sin B. This is the law of sines. ☺*

18. Use the results of two previous problems to prove that the ratio of the area of ΔABC to the area of its circumcircle is $2\cdot\sin A\cdot\sin B\cdot\sin C/\pi$. *By #2, the area of a Δ is ½(sin B·sin C/sin A)·a^2. The area of the circle is $\pi r^2 = \pi((1/2)a/\sin A)^2 = \pi a^2/4\sin^2 A$ so the*

Ratio of the areas is $$\frac{\left(\dfrac{a^2\cdot\sin B\cdot\sin C}{2\cdot\sin A}\right)}{\left(\dfrac{\pi\cdot a^2}{4\sin^2 A}\right)} = \left(\frac{a^2\cdot\sin B\cdot\sin C}{2\cdot\sin A}\right)\bullet\left(\frac{4\sin^2 A}{\pi\cdot a^2}\right) = \frac{2\sin A\cdot\sin B\cdot\sin C}{\pi}$$

19. Without using a calculator, find the exact lengths of the diagonals of the quadrilateral to the right. Use the fact that opposite angles in a cyclic quadrilateral are supplementary.



Let PR = x. By the law of cosines, $x^2 = 7^2 + 4^2 - 2\cdot7\cdot4\cdot\cos Q$. Also, by the law of cosines, $x^2 = 5^2 + 6^2 - 2\cdot5\cdot6\cdot\cos(180° - Q)$= so, $x^2 = 5^2 + 6^2 + 2\cdot5\cdot6\cdot\cos(Q) = 7^2 + 4^2 - 2\cdot7\cdot4\cdot\cos Q$.

$\cos Q(2\cdot5\cdot6+2\cdot7\cdot4) = 7^2 + 4^2 - 5^2 - 6^2$ so cos Q = $\dfrac{(7^2-6^2)+(4^2-5^2)}{2(5\cdot6+7\cdot4)} = \dfrac{13-9}{2(58)} = \dfrac{1}{29}$

so $x^2 = 5^2 + 6^2 + 2\cdot5\cdot6\cdot1/29 = 61+60/29 = 63\ 2/29 = 1829/29$. S0 x = $\sqrt{\left(\dfrac{1829}{29}\right)}$ *Similarly,*

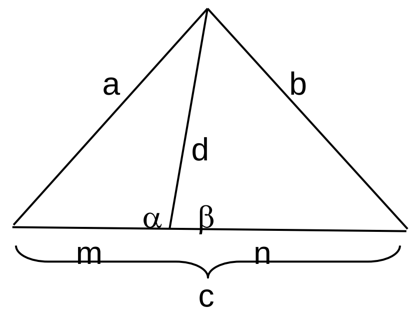
If QS = y, $y^2 = 7^2 + 5^2 - 2\cdot7\cdot5\cdot\cos P = 6^2 + 4^2 + 2\cdot6\cdot4\cdot\cos P$, so cos P =

$\dfrac{(7^2-6^2)+(5^2-4^2)}{2(4\cdot6+7\cdot5)} = \dfrac{13+9}{2(59)} = \dfrac{11}{59}$ *so $x^2 = 6^2 + 4^2 + 2\cdot6\cdot4\cdot11/59 = 52+528/59 = 3596/59$*

S0 x = $\sqrt{\left(\dfrac{3596}{59}\right)}$ *NOTE: We can check with Ptolemy's theorem: Does 7·6 + 4·5*

$= \sqrt{\left(\dfrac{1829}{29}\right)}\sqrt{\left(\dfrac{3596}{59}\right)} = \sqrt{3844} = 62.$ *Yes it does!! (I checked WITH the calculator.)*

20. Use the Law of Cosines to find a relationship between the sides of any triangle ABC, a Cevian segment **d** drawn to **c**, and the two segments created by the Cevian **m** and **n**.



A) Using the Law of Cosines,
find an expression for **a** in terms of **m** and **d**.
Solve for cos α. *$a^2 = m^2 + d^2 - 2 \cdot m \cdot d \cdot \cos\alpha$*

So $\cos\alpha = \dfrac{a^2 - m^2 - d^2}{-2md}$

B) Using the Law of Cosines,
find an expression for **b** in terms of **n** and **d**.
Solve for cos β. $b^2 = n^2 + d^2 - 2 \cdot n \cdot d \cdot \cos\beta$

So $\cos\beta = \dfrac{b^2 - n^2 - d^2}{-2nd}$

C) Using the relationship between α and β, combine these two equations. (You've done this before…)

Since $\cos\alpha = -\cos\beta$, $\cos\alpha = \dfrac{a^2 - m^2 - d^2}{-2md} = \dfrac{b^2 - n^2 - d^2}{2nd}$ so $\dfrac{a^2 - m^2 - d^2}{-m} = \dfrac{b^2 - n^2 - d^2}{n}$

D) Prove your result is equal to Stewart's theorem: $\mathbf{a^2n + b^2m = d^2c + mnc}$

So $a^2n - m^2n - d^2n = -b^2m + mn^2 + dm^2 \rightarrow a^2n + b^2m = d^2(n + m) + mn(n + m)$, *but since* $m + n = c$, $\mathbf{a^2n + b^2m = d^2c + mnc}$

21. Solve for x:

A)
$$\left(9^{\tan x}\right)^{\cos x} = \frac{1}{3}$$
$$\left(3^2\right)^{\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1}} = 3^{-1}$$
$$2\sin x = -1$$
$$\sin x = -1/2$$
$$x = \begin{cases} \dfrac{7\pi}{6} + 2\pi k \\ \dfrac{11\pi}{6} + 2\pi k \end{cases}$$

B)
$$\frac{16^{\sin x}}{2^{\cos^2 x}} = 2 \cdot 2^{\sin^2 x}$$
$$\frac{2^{4\sin x}}{2^{\cos^2 x}} = 2^{1+\sin^2 x}$$
$$4\sin x - (1 - \sin^2 x) = 1 + \sin^2 x$$
$$4\sin x - 1 = 1$$
$$4\sin x = 2$$
$$\sin x = 1/2$$
$$x = \begin{cases} \dfrac{\pi}{6} + 2\pi k \\ \dfrac{5\pi}{6} + 2\pi k \end{cases}$$