Scalar and Vector Projections

Accelerated Precalculus

February 12, 2019

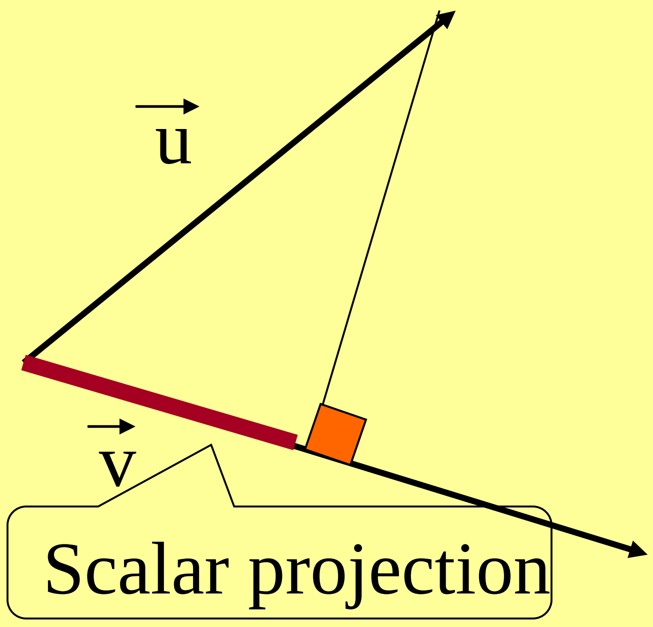
One-Minute Question

- Evaluate:

  $(3\vec{i} - 7\vec{j}) \cdot (4\vec{i} + \vec{j})$

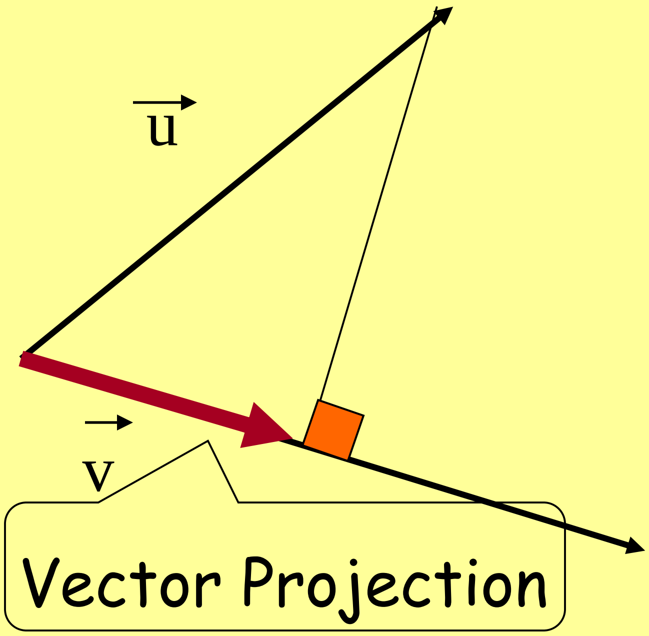
Scalar Projections

- A scalar projection of $\vec{\mathbf{u}}$ onto $\vec{\mathbf{v}}$ is the length of the shadow that $\vec{\mathbf{u}}$ casts on $\vec{\mathbf{v}}$.



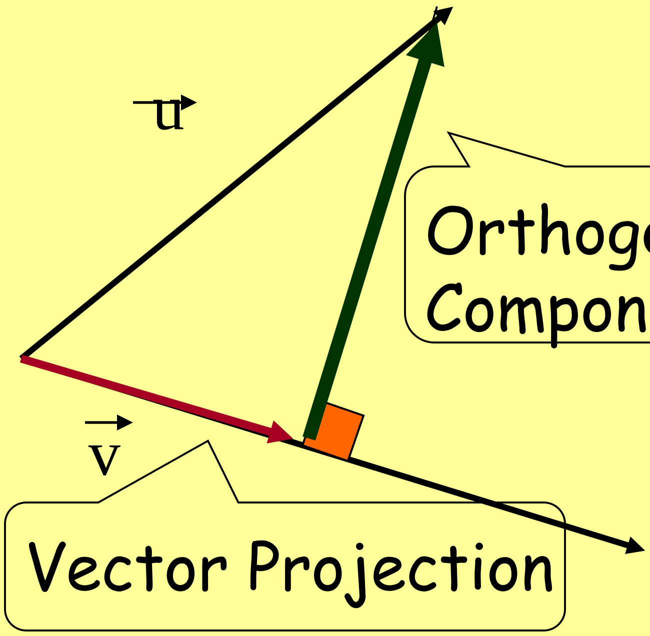
Vector Projections

- A vector projection of $\vec{u}$ onto $\vec{v}$ is the vector in the same direction as $\vec{v}$, whose length is the scalar projection of $\vec{u}$ on $\vec{v}$.

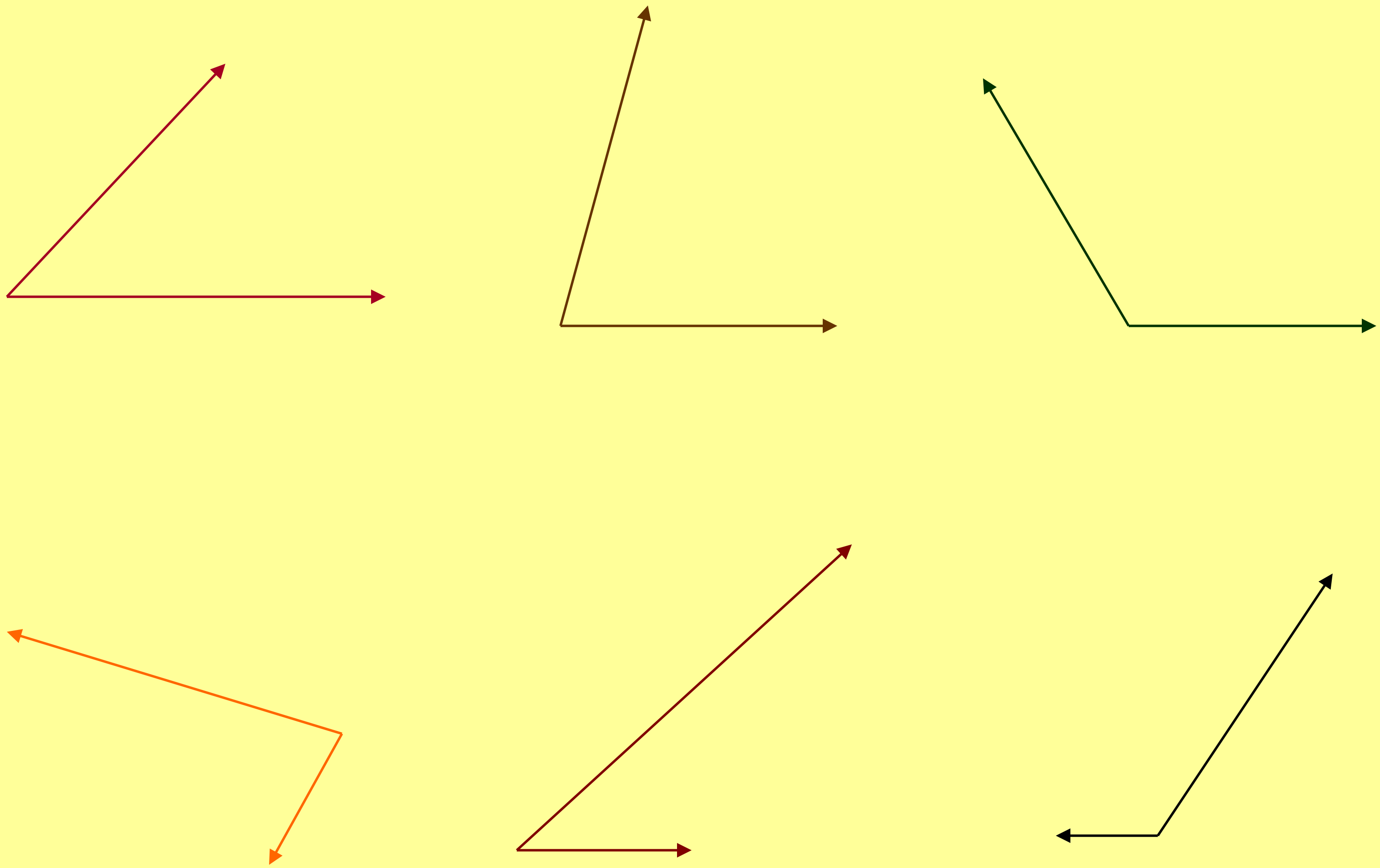


Vector Projections

- A vector component of $\vec{u}$ orthogonal to $\vec{v}$ is the vector perpendicular to $\vec{v}$, that adds to the vector projection of $\vec{u}$ on $\vec{v}$ to create $\vec{u}$.



- Now you try these!!



So What Happens If We Have Angles and Lengths?

- How long is the scalar projection of $\vec{\mathbf{u}}$ onto $\vec{\mathbf{v}}$?

- Shouldn't it be: $|\vec{u}|\cos\theta$
  if θ is the angle between vectors

So try these!

- Find the scalar projection of $\vec{\mathbf{r}}$ on $\vec{\mathbf{s}}$ if $|\vec{\mathbf{r}}| = 10$, $|\vec{\mathbf{s}}| = 16$, and the angle between them is 140°.

- $10\cos 140° = -7.66$

- Find the length of the orthogonal component of $\vec{\mathbf{r}}$ on $\vec{\mathbf{s}}$.

- $(-7.66)^2 + |\vec{\mathbf{o}}|^2 = 10^2$

- So $|\vec{\mathbf{o}}| = 6.428$

So What Happens If We Have Coordinates?

- How long is the scalar projection of $\vec{\mathbf{u}}$ onto $\vec{\mathbf{v}}$?

- Shouldn't it be: $|\vec{\mathbf{u}}| \cos \theta$ if θ is the angle between vectors
- So isn't that $|\vec{\mathbf{u}}| \dfrac{(\vec{\mathbf{u}} \cdot \vec{\mathbf{v}})}{|\vec{\mathbf{u}}|\,|\vec{\mathbf{v}}|} = \dfrac{(\vec{\mathbf{u}} \cdot \vec{\mathbf{v}})}{|\vec{\mathbf{v}}|}$

Try This One!

- Find the scalar projection, the vector projection, and the orthogonal component of:

$(6\vec{i} + 7\vec{j})$ onto $(5\vec{i} - 12\vec{j})$

The scalar projection is:

- $6 \cdot 5 + 7 \cdot (-12) = -54/13$

The vector projection is:

$-54/13(5/13\vec{i} - 12/13\vec{j}) =$

$-270\vec{i}/169 + 648\vec{j}/169$

The orthogonal component is:

$(6\vec{i} + 7\vec{j}) -$

$(-270\vec{i}/169 + 648\vec{j}/169) =$

$1284\vec{i}/169 - 535\vec{j}/169$