More Algebraic (And Older) Parametric Worksheet Name________________________________

1. A cricket ball is hit with a velocity of 12 m/s at an angle with the horizontal whose tangent is 4/3. Find:

   a) The greatest height reached.

   b) The time of flight.

   c) The horizontal distance traveled.

2. A ball is projected so that its horizontal range is 45meters and it passes through a point 22 meters horizontally from and 11 meters vertically above the point of projection. Find the angle of projection and the speed of projection.

3. A ball thrown horizontally with speed V from a point H meters above the ground lands at a horizontal distance D meters from the point of release. Find:

   a) V if D = 3 and H = 1.

   b) H if V = 10 and D = 20.

4. A golfer hits a ball from a point on a level fairway and 2 seconds later it strikes the fairway 50 m away. Find:

   a) The initial velocity and angle of projection.

   b) The maximum height of the ball above the fairway.

5. A batsman hits a cricket ball "off his toes" towards a fieldsman who is 65 m away. The ball reaches a maximum height of 4.9 m and the horizontal component of its velocity is 28 m/s. Find the constant speed with which the fieldsman must run forward, starting at the instant the ball is hit, in order to catch the ball at a height of 1.3 m above the ground.

6. A football kicked at 16 m/s, just passes over a crossbar 4 m high and 16 m away. Show that, if θ if the angle of projection, $4.9\tan^2\theta - 16\tan\theta + 8.9 = 0$.

Worksheet Solutions:

1. Since the cricket ball is an object in flight, $x(t) = 12(3/5)t$ and $y(t) = 12(4/5)t - 4.9t^2$

A) Use the y(t) equation and $t_{max} = -b/(2a) = -12(4/5)/(2(-4.9) \approx .9796$ seconds, so $Y(.9796) \approx 4.702$ meters.

B) Find t when y(t) = 0, so factoring $y(t) = t(12(4/5) - 4.9t) \Rightarrow t \approx 1.959$ seconds.

C) Find x when $t \approx 1.959$ seconds $\Rightarrow x \approx 14.106$ meters

2. You know that $x = v_0\cos\theta \cdot t$ and that $y = v_0\sin\theta \cdot t - 4.9t^2$.

A) This graph passes through the point (45, 0), so substituting that point in, we get that $45 = v_0\cos\theta \cdot t$ and $v_0\sin\theta \cdot t = 4.9t^2 \Rightarrow v_0\sin\theta = 4.9t \Rightarrow t = v_0\sin\theta/4.9$. Substituting, we get $45 = v_0\cos\theta \cdot v_0\sin\theta/4.9 \Rightarrow$

$v_0^2 = 45 \cdot 4.9/(\cos\theta \cdot \sin\theta) \Rightarrow v_o = \sqrt{(45 \cdot 4.9)/(\cos\theta \cdot \sin\theta)}$.

B) The graph also passes through (22, 11) at a different time, so if $22 = v_0 \cos\theta \cdot t$, then $t = \dfrac{22}{\cos\theta}\sqrt{\dfrac{\cos\theta \cdot \sin\theta}{45 \cdot 4.9}}$.

Meanwhile y yields: $11 = \sqrt{\dfrac{(45 \cdot 4.9)}{(\cos\theta \cdot \sin\theta)}} \cdot \sin\theta \cdot \dfrac{22}{\cos\theta}\sqrt{\dfrac{\cos\theta \cdot \sin\theta}{45 \cdot 4.9}} - 4.9 \cdot \left(\dfrac{22}{\cos\theta}\sqrt{\dfrac{\cos\theta \cdot \sin\theta}{45 \cdot 4.9}}\right)^2 =$

$11 = 22\tan\theta - 4.9 \cdot \left(\dfrac{22^2}{\cos^2\theta}\dfrac{\cos\theta \cdot \sin\theta}{45 \cdot 4.9}\right)$. Simplifying further, we get $11 = 22\tan\theta - (22^2/45)\tan\theta$. So $\tan\theta = .9783$, so $\theta = 44.37°$, leaving $v_o = 21.00$ m/sec. Therefore parametric equations of this motion are:

$x = 21 \cdot t \cdot \cos 44.37º$ and $y = 21 \cdot t \cdot \sin 44.37º - 4.9t^2$

3. $x = Vt$ and $y = -4.9t^2 + H$ so

a) If D = 3 and H = 1, $4.9t^2 = 1 \Rightarrow t = 0.45176$ seconds, so $3 = V*0.45176 \Rightarrow V = 6.641$ m/sec.

b) If V = 10 and D = 20, $20 = 10t \Rightarrow t = 2$, $H = 4.9(2)^2 = 19.6$ meters.

4. a) $50 = v_0\cos\theta \cdot 2 \Rightarrow v_0\cos\theta = 25 \Rightarrow \Rightarrow v_0 = 25/\cos\theta$. And $0 = v_0\sin\theta \cdot 2 - 4.9 \cdot 2^2 \Rightarrow (25/\cos\theta)\sin\theta \cdot 2 = 4.9 \cdot 4$, so $\tan\theta = 9.8/25$ so $\theta = 21.41°$ and $v_0 = 25/\cos\theta = 26.85$ m/sec.

b) Since y = 0 at t = 0 and t = 2, by symmetry, the maximum height occurs at t = 1, so y(1) = 4.9 meters.

5. We know that $x = 28 \cdot t$ and that when $t = v_0\sin t/9.8$, $y = 4.9$, so let $v_0\sin\theta = k$ and we see that $v_0\sin\theta \cdot v_0\sin t/9.8 - 4.9(v_0\sin t/9.8)^2 = 9.8 \Rightarrow k^2/9.8 - 4.9k^2/9.8^2 = 4.9 \Rightarrow k = 9.8$ (seriously, try it!), **so $x = 28 \cdot t$ and $y = 9.8t - 4.9t^2$.** If y = 1.3, then $-4.9t^2 + 9.8t - 1.3 = 0 \Rightarrow t = 0.14286$ or $t = 1.8571$ seconds (and I think this time we need the larger value). So x(1.8571) = 52 and the fieldsman's speed must be(65 – 52)/1.8571 = 7 m/s.

6. Parametric equations will be $x = 16t\cos\theta$ and $y = 16t\sin\theta - 4.9t^2$. If $16t\cos\theta = 16$ when $4 = 16t\sin\theta - 4.9t^2$, $t = 1/\cos\theta$ and $4 = 16\sin\theta/\cos\theta - 4.9(\sec^2\theta) \Rightarrow 4 = 16\tan\theta - 4.9(1 + \tan^2\theta) \Rightarrow 4 = 16\tan\theta - 4.9 - 4.9\tan^2\theta \Rightarrow$

$4.9\tan^2\theta - 16\tan\theta + 8.9 = 0$