Mathematical Induction Proofs:

**1. Prove that 1 + 2 + 3 + … + n = ½(n2 + n)**

Let n = 1. Then 1 = 1 and ½ (12 + 1) = ½ (2) = 1so the proof is anchored.

Assume our statement holds for some k∈N. Then 1 + 2 + 3 + … + k = ½(k2 + k)

We will show that 1 + 2 + 3 + … + k + (k + 1) = ½((k + 1)2 + (k + 1))

 ½ (k2 + k) + (k + 1) = ½ (k2 + k + 2k + 2) =½ (k2 + k + 2k + 1 + 1) = ½ ((k2 + 2k + 1) + k + 1)

 = ½ (( k + 1)2 + (k + 1))

Since the kth term implies the (k + 1)st  term, all k∈N terms hold.

Therefore, by the Principle of Mathematical Induction, 1 + 2 + 3 + … + n = ½(n2 + n).

**2. Prove that 2 + 5 + 8 + … + (3n – 1) = **

Let n = 1. Then (31 – 1) = 2 and  so the proof is anchored.

We can assume this holds for some k∈N, meaning that 2 + 5 + 8 + … + (3k – 1) = 

We must show that that 2 + 5 + 8 + … + (3k – 1) + (3(k + 1) – 1) = 

 + (3(k + 1) – 1) = + (3k + 3 – 1) =

 = 

Since the kth term implies the (k + 1)st  term, all k∈N terms hold.

Therefore, by the Principle of Mathematical Induction, 2 + 5 + 8 + … + (3n – 1) = 

3. **Prove that 1 + 3 + 5 + … + 2n – 1 = n2**

Let n = 1. 21 – 1 = 1 and 11 = 1 so the first case holds.

Therefore this statement must be true for some k∈N, meaning that 1 + 3 + 5 + … + 2k – 1 = k2.

This implies that 1 + 3 + 5 + … + (2k – 1) + (2(k+1) – 1) = k2 + (2(k+1) – 1) = k2 + 2k + 2 – 1 = k2 + 2k + 1 =

(k + 1)2. So if the kth  holds, then the (k+1)st case holds. So by the Principle of Mathematical

Induction, 1 + 3 + 5 + … + 2n – 1 = n2.

4**. Prove that 3 + 32 + 33 + … + 3n = **

Let n = 1. Then 3 = 3 and  so the proof is anchored.

We can assume this holds for some k∈N, meaning that 3+ 32 + 33 + … + 3k = 

We must show that that 3+ 32 + 33 + … + 3k + 3k+1 = 

  + 3k+1 =  +  = 

Since the kth term implies the (k + 1)st  term, all k∈N terms hold.

Therefore, by the Principle of Mathematical Induction, 3 + 32 + 33 + … + 3n = ****

5. Prove: **: This means: 12 + 22 + 32 + … + n2 =**

Let n = 1. Then 1 = 1 and  so the proof is anchored.

We can assume this holds for some k∈N, meaning that 12 + 22 + 32 + … + k2 =

This means that 12 + 22 + 32 + … + k2 + (k+1)2 = + (k+1)2 = =





Since the kth term implies the (k + 1)st  term, all k∈N terms hold.

Therefore, by the Principle of Mathematical Induction, 12 + 22 + 32 + … + n2 = ****

6. Prove: **1 + 2 + 22 + 23 + … + 2n – 1 = 2n – 1**

Let n = 1. 21-1 = 20 = 1 and 21 – 1 = 1 so the first case holds.

Therefore this statement must be true for some k∈N, meaning that 1 + 2 + 22 + 23 + … + 2k – 1 = 2k – 1.

This implies that 1 + 2 + 22 + 23 + … + 2k – 1 + 2(k + 1) - 1 = 2k – 1 + 2(k + 1) – 1 = 2k – 1 + 2k = 22k – 1 = 2k + 1 – 1.

So if the kth  holds, then the (k+1)st case holds. So by the Principle of Mathematical

Induction, **1 + 2 + 22 + 23 + … + 2n – 1 = 2n – 1.**