Linear Combinations Theorem Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

A Proof! Period\_\_\_\_\_Date\_\_\_\_\_\_\_\_\_\_

Write each of the following as a single sinusoid (preferably a cosine equation) with a phase change in either degrees or radians, as indicated.

1. (radians) f(x) = 3cos x + 4sin x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2. (degrees) g(θ) = 5cos θ + 12sin θ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

3. (radians) h(x) = 5cos x - 4sin x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

4. (degrees) g(θ) = 5sin θ - 12cos θ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Describe how you are finding the new amplitude of the problems above.

Describe how you are finding the new phase shift of the problems above.

The Proof!

Consider f(θ) = Ccos θ + Dsin θ.

1. (Multiply f(θ) by .)

2. (Distribute only the denominator of this fraction.)

3. (Rearrange each of the two terms so that

the algebraic part creates one fraction

that is the coefficient of a trig function.)

4. (Realizing that each coefficient could also be a trig

expression, draw a triangle in which the coefficient

of the cosine is itself the cosine of some angle **K** and

the coefficient of the sine is the sine of that angle.)

5. (Replace the algebraic fractions with the expressions

“sin **K**” and “cos **K**”.)

6. (Use a trig identity to rewrite the above expression.)

7. What will the tangent of **K** be?

8. Can we say that K = tan-1(D/C)? Why or why not?

Test your theory with these problems:

Write as a single sinusoid with a phase change. Check your answers by graphing.

\* These don’t require a calculator.

1. y = 7cos x + 3sin x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2. f(θ) = 8cos θ – sin θ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

3. y(x) = 3sin x + 4cos x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\*4. g(x) = sin x – cos x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

5. y = cos(3x) – sin(3x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What effect does the 3 have on your work?

Solve the following equations algebraically if 0 < x < 2π or 0° < θ < 360°

\*6. 2cos x + 2sin x = 

\*7. sin θ – cos θ = 

8. 6cos x – 3sin x = 8

9. -12cos θ – 5sin θ = 10

Write each of the following as a linear combination of cosine and sine.

\*10. 8cos(θ – 120°) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\*11. 4cos(x + 5π/4) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\*12. 6sin(x – π/2) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

13. 4cos(x + 27°) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

14. (Think about this one.) 2cos (2x – π) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

A sinusoidal function can be written in terms of either sine or cosine, but we have decided to write these functions as cosine functions because the phase change is more obvious. What would we need to change in order to write these sums as sine functions?