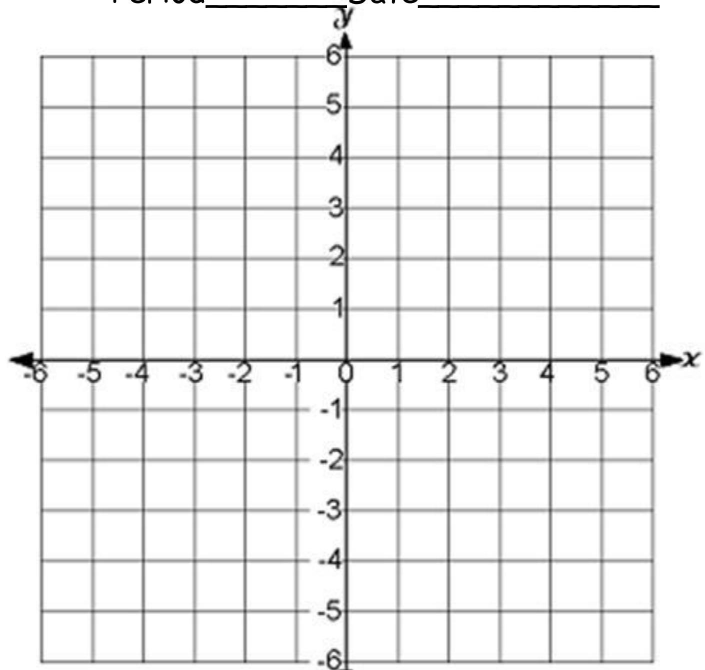
Inverses of Trig Functions

Name_________________________

Period_______Date_____________

I. On the graph to the right, sketch a fairly accurate graph of y = sin x.




1. Describe the relationship between a graph and its inverse.

2. Describe the relationship between specific points on a graph and points on its inverse.

3. Use this information to write at least 6 points on a graph of the inverse of your sine function.

4. Plot the points from problem #3 and connect them to form a smooth inverse graph.
5. Will this inverse be a function?  Why or why not?

One convention is to call **this relation** y = arcsin x. It's domain is $-1 \le x \le 1$ and its range is all reals. The **relation** y = arcsinx has many values when $x = \frac{1}{2}$. List at least three .

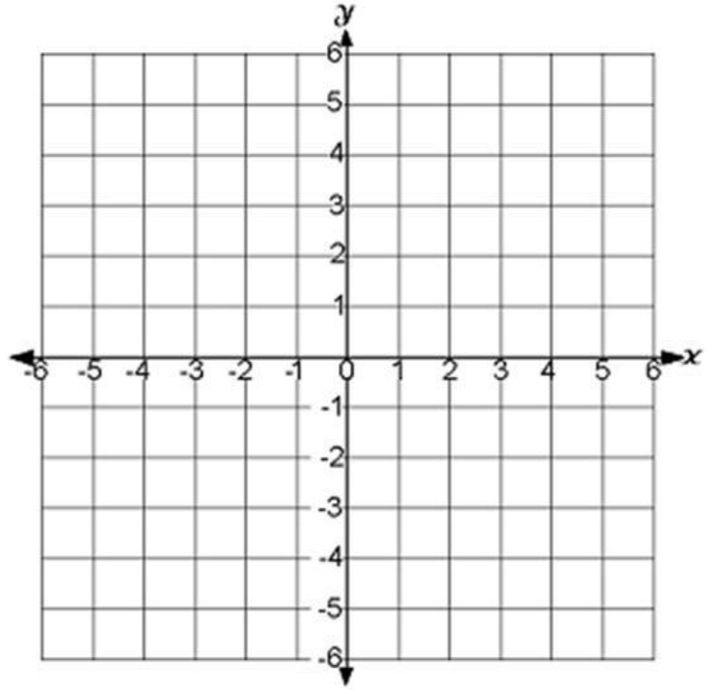
6. If you wanted to create an **inverse function** for the sine of x, you would have to limit the range of the arcsine relation without limiting the domain. How would you limit the range of the arcsine relation to create a function?

   _______ $\le y \le$ ________

7. Graph f(x) = sin (x) and f(x) = $\sin^{-1}$ (x) on your calculator with a Zoom Decimal window. Does y = $\sin^{-1}$ (x) appear to have the same range as your limits?

   Check by finding $\sin^{-1}(-1)$ _________ and $\sin^{-1}(1)$. ___________ and then, in another color, graph y = $\sin^{-1}(x)$ on the graph above. **Although our text does not do this, my convention will be to use f(x) = $\text{Sin}^{-1}$(x) or f(x) = Arcsin (x) to denote the function.**

II. 1. Graph y = cos x to the right.




2. Graph the inverse **relation**: y = arccos x.

3. How would you limit the range of the arcos x to create the **function** $g(x) = Cos^{-1}(x)$?

_____ $\le y \le$ _______

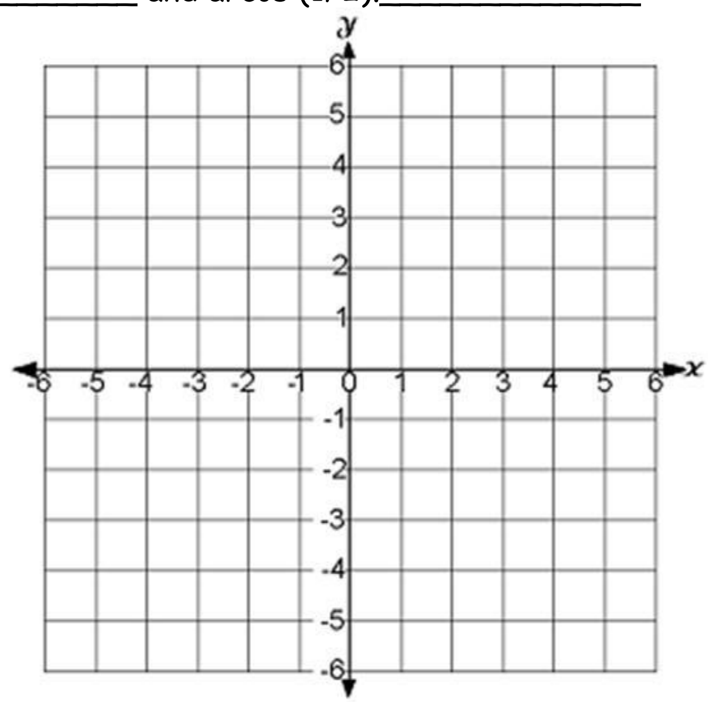
4. Graph y1 = cos (x) and $y2 = Cos^{-1}(x)$ on your calculator to confirm your prediction.

5. If f(x) = cos (x) graphs (angle, ratio), then $f^{-1}(x)$ graphs (________, ________)

6. Evaluate cos (π/3) = _________

cos (5π/3) = ________, $Cos^{-1}(1/2)$ = ________ and arcos (1/2).______________

III. 1. Sketch a good graph of y = tan (x).




2. Now sketch the graph of the inverse relation arctan in a light color.

3. What cycle would you predict to be the graph of $f(x) = Tan^{-1}(x)$? Check your prediction with your calculator and graph that in a darker color.

4. Domain of y = tan (x).___________

Range of y = tan (x). _____________

Domain of y = arctan(x). __________

Range of y = arctan(x). ___________

Domain of $y = Tan^{-1}(x)$.___________

Range of $y = Tan^{-1}(x)$. _____________

IV. Evaluate the following:

1. $Sin^{-1}(-1/2)$ = __________, $Cos^{-1}(-1/2)$ = ___________, $Tan^{-1}(-1)$ = _____________

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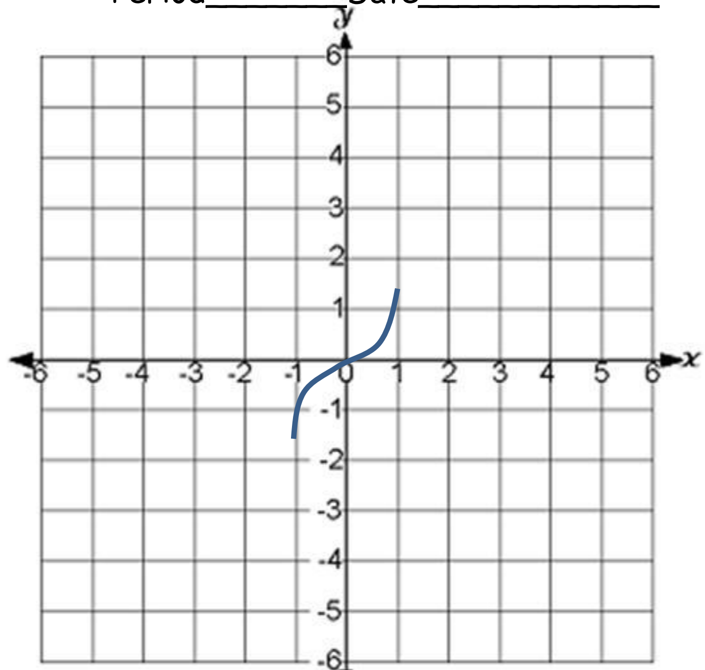



1. Describe the relationship between a graph and its inverse.

   It is reflected over the line y = x.

2. Describe the relationship between specific points on a graph and points on its inverse.

   The x and y values are switched.

3. Use this information to write at least 6 points on a graph of the inverse of your sine function.

4. Plot the points from problem #3 and connect them to form a smooth inverse graph.

5. Will this inverse be a function? Why or why not?

   No. Infinitely many angles have the same sine value.

   One convention is to call **this relation** y = arcsin x. It's domain is $-1 \le x \le 1$ and its range is all reals. The **relation** y = arcsinx has many values when $x = \frac{1}{2}$. List at least three .

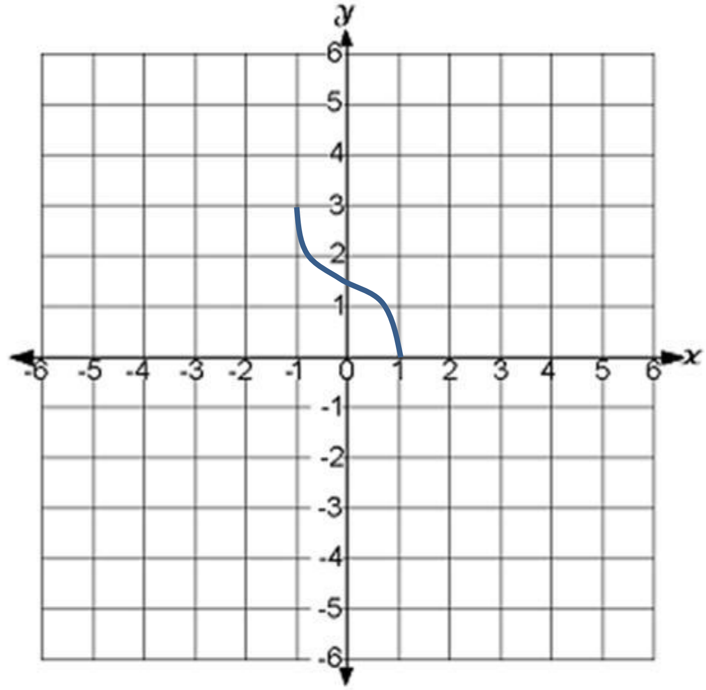
6. If you wanted to create an **inverse function** for the sine of x, you would have to limit the range of the arcsine relation without limiting the domain. How would you limit the range of the arcsine relation to create a function?

   __-π/2_ < y < _π/2__

7. Graph f(x) = sin (x) and f(x) = $\sin^{-1}$ (x) on your calculator with a Zoom Decimal window. Does y = $\sin^{-1}$ (x) appear to have the same range as your limits?

   Check by finding $\sin^{-1}$(-1) _-π/2__ and $\sin^{-1}$(1). __π/2____ and then, in another color, graph y = $\sin^{-1}$(x) on the graph above. **Although our text does not do this, my convention will be to use f(x) = $\text{Sin}^{-1}$(x) or f(x) = Arcsin (x) to denote the function.**

II. 1. Graph y = cos x to the right.




2. Graph the inverse **relation**: y = arccos x.

3. How would you limit the range of the arcos x to create the **function** $g(x) = Cos^{-1}(x)$?

__0 _ < y < _π___

4. Graph y1 = cos (x) and $y2 = Cos^{-1}(x)$ on your calculator to confirm your prediction.

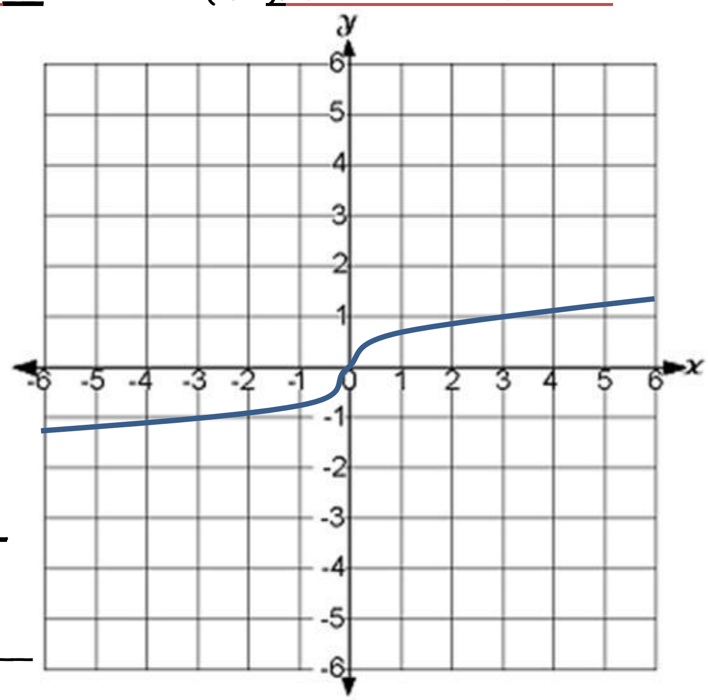
5. If f(x) = cos (x) graphs (angle, ratio), then $f^{-1}$ (x) graphs (ratio, _angle)

6. Evaluate cos (π/3) = _1/2____

If k∈ I

cos (5π/3) = ___1/2_, $Cos^{-1}(1/2)$ = _π/3__ and arcos (1/2) π/3+2πk or 5π/3+2πk

III. 1. Sketch a good graph of y = tan (x).




2. Now sketch the graph of the inverse relation arctan in a light color.

3. What cycle would you predict to be the graph of $f(x) = Tan^{-1}(x)$? Check your prediction with your calculator and graph that in a darker color.

4. Domain of y = tan (x).__x ≠π/2 + πk__

Range of y = tan (x). _All Reals_____

Domain of y = arctan(x). _All Reals_____

Range of y = arctan(x). _x ≠π/2 + πk__

Domain of $y = Tan^{-1}(x)$._All Reals_____

Range of $y = Tan^{-1}(x)$. ___-π/2 < y < π/2__

IV. Evaluate the following:

1. $Sin^{-1}(-1/2)$ = ____-π/6____, $Cos^{-1}(-1/2)$ = ____2π/3____, $Tan^{-1}(-1)$ = ___-π/4______

Inverses of Trig Functions
Exercises

Name________________________
Period_______Date____________

For the first 4 problems, sketch both graphs on the same set of axes. Label at least 3 points on each graph (endpoints if possible). Use your own paper.

1. $y1(x) = Cos^{-1}(x)$ and $y2(x) = 3Cos^{-1}(x)$

2. $y1 = Sin^{-1}(x)$ and $y2 = Sin^{-1}(3x)$

3. $f(x) = Tan^{-1}(x)$ and $g(x) = \pi + Tan^{-1}(x)$

4. $y = Cos^{-1}(x)$ and $y = Cos^{-1}(x - 2)$

Sketch each of the following without a calculator. Check your graph with one.

5. $y = \frac{\pi}{3} + \frac{2}{3}Sin^{-1}(x-1)$

6. $y = 4Cos^{-1}\left(\frac{1}{2}(x-1)\right)$

7. $y = \pi + 2Tan^{-1}(x-3)$

8. $y = \frac{\pi}{2} - Sin^{-1}(x)$

9. $y = 2\pi + 3Cos^{-1}\left(\frac{1}{3}(x+2)\right)$

10. $y = 5Tan^{-1}(2(x-1))$

Find the exact value(s) of each of the following:

1. $Sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. $Sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

3. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

4. $\arcsin\left(\frac{-\sqrt{3}}{2}\right)$

5. $\text{Sin}^{-1}\left(\frac{\sqrt{2}}{2}\right)$

6. $\text{Sin}^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

7. $\arcsin\left(\frac{\sqrt{2}}{2}\right)$

8. $\arcsin\left(\frac{-\sqrt{2}}{2}\right)$

9. $\text{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right)$

10. $\text{Cos}^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

11. $\arccos\left(\frac{\sqrt{3}}{2}\right)$

12. $\arccos\left(\frac{-\sqrt{3}}{2}\right)$

13. $\text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right)$

14. $\text{Cos}^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

15. $\arccos\left(\frac{\sqrt{2}}{2}\right)$

16. $\arccos\left(\frac{-\sqrt{2}}{2}\right)$

17. $\text{Sin}^{-1}\left(\cos\frac{\pi}{2}\right)$

18. $\text{Sin}^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)$

19. $\sin\left(\arcsin\left(\frac{\sqrt{3}}{2}\right)\right)$

20. $\cos\left(\text{Cos}^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right)$

21. $\sin\left(\arccos\left(\frac{3}{5}\right)\right)$

22. $\cos\left(\text{Sin}^{-1}\left(\frac{-5}{13}\right)\right)$

23. $\cos\left(\text{Sin}^{-1}\left(\frac{\sqrt{5}}{7}\right)\right)$

24. $\tan\left(\text{Cos}^{-1}\left(\frac{-8}{17}\right)\right)$

25. $\cos\left(\text{Tan}^{-1}\left(\frac{-4}{3}\right)\right)$

26. $\tan\left(\text{Sin}^{-1}\left(\frac{7}{25}\right)\right)$

27. $\tan\left(\text{Cos}^{-1}(x)\right)$

28. $\sin\left(\text{Cos}^{-1}(\theta)\right)$

29. $\cos\left(\text{Tan}^{-1}(x)\right)$

30. $\cos\left(\text{Sin}^{-1}(\theta)\right)$

Inverses of Trig Functions
Exercises

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For the first 4 problems, sketch both graphs on the same set of axes. Label at least 3 points on each graph (endpoints if possible). Use your own paper.

1. $y1(x) = Cos^{-1}(x)$ and $y2(x) = 3Cos^{-1}(x)$ y1: {(1, 0), (0, π/2), (-1, π)} y2: { (1, 0), (0, 3π/2), (-1, 3π)}

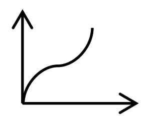
2. $y1 = Sin^{-1}(x)$ and $y2 = Sin^{-1}(3x)$ Y1: {(-1, -π/2), (0, 0), (1, π/2)} y2: {(-1/3, -π/2), (0, 0), (1/3, π/2)}

3. $f(x) = Tan^{-1}(x)$ and $g(x) = \pi + Tan^{-1}(x)$ f(x): {(0, 0), (1, π/4), (-1, -π/4)} g(x): {(0, π), (1, 5π/4), (-1, 3π/4)}

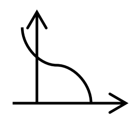
4. $y = Cos^{-1}(x)$ and $y = Cos^{-1}(x - 2)$ y1: {(1, 0), (0, π/2), (-1, π)} y2: { (3, 0), (2, 3π/2), (1, 3π)}

Sketch each of the following without a calculator. Check your graph with one.

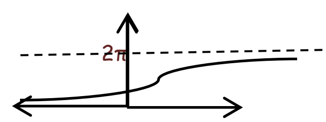
5. $y = \frac{\pi}{3} + \frac{2}{3}Sin^{-1}(x-1)$ Points: $\left\{(0,0),\left(1,\frac{\pi}{3}\right),\left(2,\frac{2\pi}{3}\right)\right\}$



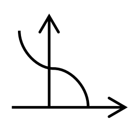
6. $y = 4Cos^{-1}\left(\frac{1}{2}(x-1)\right)$ Points: $\{(3,0),(1,2\pi),(-1,4\pi)\}$



7. $y = \pi + 2Tan^{-1}(x-3)$ Points: $\left\{\left(4,\frac{3\pi}{2}\right),(3,\pi),\left(2,\frac{\pi}{2}\right)\right\}$

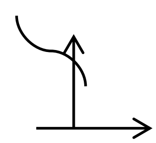



8. $y = \frac{\pi}{2} - Sin^{-1}(x)$ Points: $\left\{(1,0),\left(0,\frac{\pi}{2}\right),(-1,\pi)\right\}$

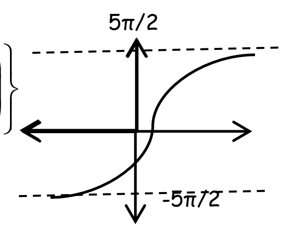


(This is a $y = cos^{-1}(x)$.)

9. $y = 2\pi + 3Cos^{-1}\left(\frac{1}{3}(x+2)\right)$ Points: $\left\{(1,2\pi),\left(-2,\frac{7\pi}{2}\right),(-5,5\pi)\right\}$



10. $y = 5Tan^{-1}(2(x-1))$ Points: $\left\{(1,0),\left(\frac{3}{2},\frac{5\pi}{4}\right),\left(\frac{1}{2},\frac{-5\pi}{4}\right)\right\}$




Find the exact value of each of the following:

1. $Sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

2. $Sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-\pi}{3}$

3. $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \left\{\frac{\pi}{3} + 2\pi k, \frac{2\pi}{3} + 2\pi k\right\}$

4. $\arcsin\left(\frac{-\sqrt{3}}{2}\right) = \left\{\frac{-\pi}{3} + 2\pi k, \frac{4\pi}{3} + 2\pi k\right\}$

5. $\mathrm{Sin}^{-1}\left(\frac{\sqrt{2}}{2}\right)=\frac{\pi}{4}$

6. $\mathrm{Sin}^{-1}\left(\frac{-\sqrt{2}}{2}\right)=\frac{-\pi}{4}$

7. $\arcsin\left(\frac{\sqrt{2}}{2}\right)=\left\{\frac{\pi}{4}+2\pi k,\ \frac{3\pi}{4}+2\pi k\right\}$

8. $\arcsin\left(\frac{-\sqrt{2}}{2}\right)=\left\{\frac{-\pi}{4}+2\pi k,\ \frac{5\pi}{4}+2\pi k\right\}$

9. $\mathrm{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$

10. $\mathrm{Cos}^{-1}\left(\frac{-\sqrt{3}}{2}\right)=\frac{5\pi}{6}$

11. $\arccos\left(\frac{\sqrt{3}}{2}\right)=\left\{\frac{\pi}{6}+2\pi k,\ \frac{-\pi}{6}+2\pi k\right\}$

12. $\arccos\left(\frac{-\sqrt{3}}{2}\right)=\left\{\frac{5\pi}{6}+2\pi k,\ \frac{-5\pi}{6}+2\pi k\right\}$

13. $\mathrm{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right)=\frac{\pi}{4}$

14. $\mathrm{Cos}^{-1}\left(\frac{-\sqrt{2}}{2}\right)=\frac{3\pi}{4}$

15. $\arccos\left(\frac{\sqrt{2}}{2}\right)=\left\{\frac{\pi}{4}+2\pi k,\ \frac{-\pi}{4}+2\pi k\right\}$

16. $\arccos\left(\frac{-\sqrt{2}}{2}\right)=\left\{\frac{3\pi}{4}+2\pi k,\ \frac{-3\pi}{4}+2\pi k\right\}$

17. $\mathrm{Sin}^{-1}\left(\cos\frac{\pi}{2}\right)=0$

18. $\mathrm{Sin}^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)=\frac{-\pi}{6}$

19. $\sin\left(\arcsin\left(\frac{\sqrt{3}}{2}\right)\right)=\frac{\sqrt{3}}{2}$

20. $\cos\left(\mathrm{Cos}^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right)=\frac{-\sqrt{3}}{2}$

21. $\sin\left(\arccos\left(\frac{3}{5}\right)\right)=\frac{4}{5}$

22. $\cos\left(\mathrm{Sin}^{-1}\left(\frac{-5}{13}\right)\right)=\frac{12}{13}$

23. $\cos\left(\mathrm{Sin}^{-1}\left(\frac{\sqrt{5}}{7}\right)\right)=\frac{\sqrt{44}}{7}$

24. $\tan\left(\mathrm{Cos}^{-1}\left(\frac{-8}{17}\right)\right)=\frac{-15}{8}=-\frac{15}{8}=\frac{15}{-8}$

25. $\cos\left(\mathrm{Tan}^{-1}\left(\frac{-4}{3}\right)\right)=\frac{3}{5}$

26. $\tan\left(\mathrm{Sin}^{-1}\left(\frac{7}{25}\right)\right)=\frac{7}{24}$

27. $\tan\left(\mathrm{Cos}^{-1}(x)\right)=\frac{\sqrt{1-x^2}}{x}$

28. $\sin\left(\mathrm{Cos}^{-1}(\theta)\right)=\sqrt{1-\theta^2}$

29. $\cos\left(\mathrm{Tan}^{-1}(x)\right)=\frac{1}{\sqrt{1+x^2}}$

30. $\cos\left(\mathrm{Sin}^{-1}(\theta)\right)=\sqrt{1-\theta^2}$