Selected Answers to “Don’t We Just Love Probability”

I. 1. N objects can be partitioned into k subsets C(n+k-1, n) ways.

 2. C(20 + 5 – 1, 20) = 10626

 3. 38 = 6561 (I just can’t see this as a partition! Puppies need special families. Order

 will matter to the pups.)

 4. C(7, 5) – 3 = 18 5. C(7 + 4, 7) = 330

 6. C( 4 + 6, 4) = 210 7∙7∙7∙7 = 74 = 2401

 7. C(12 + 11, 12) = 1352078

II. 1. Select 7 random integers from 1 to 4. Each time a 1 appears, let that represent a sunny day, but let the numbers 2 – 4 represent rain. Count the number of rainy days.

{4, 4, 1, 2, 1, 4, 1} ⇒ It rains 4 days.

{3, 1, 4, 4, 2, 2, 1} ⇒ It rains 5 days.

{1, 3, 1, 2, 2, 4, 1} ⇒ It rains 4 days. According to the simulation, it rains, on average

{4, 3, 1, 4, 3, 2, 1} ⇒ It rains 5 days. 4.6 (or 5) days.

{4, 4, 1, 1, 3, 4, 3} ⇒ It rains 5 days.

2. Select 7 random integers from 1-100. For the first number, let 1-50 represent a rainy day and let 51-100 represent a rainy day. For each day thereafter, first look at the previous day. If the previous day is rainy, then let 1 – 75 represent another rainy day, but let 76-100 represent a sunny day. If the previous day is sunny, let 1-40 represent a rainy day, and let 41-100 represent a sunny day. Count the number of rainy days.

{32, 9, 100, 20, 40, 12, 25} ⇒ R, R, S, R, R, R, R or 6 rainy days

{86, 38, 60, 61, 8, 2, 76} ⇒ S, R, R, R, R, R, S or 5 rainy days

{70, 71, 89, 38, 12, 46, 99} ⇒ S, S, S, R, R, R, S or 3 rainy days, etc.

5. C(10, 7)(.5)7(.5)3 + C(10, 8)(.5)8(.5)2 + C(10, 9)(.5)9(.5)1 + C(10, 10)∙(.5)10 = 11/64 ≈ .171875

6. .7(.7)∙2 + (.7)(.3)∙1 = 1.19

III. P = 1/6 + 2/6∙P ⇒ P = ¼ It is not a fair game, because you win too often for the payoff. (In the end you make money.)

IV. I’ll calculate the expected winnings and then subtract $1 at the end.

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| Options | Probability | Payoff | Probability X Payoff |
| Win $1000 | 5/100000 | $1000 | .05 |
| Win $500 | 25/100000 | $500 | .125 |
| Win $100 | 50/100000 | $100 | .05 |
| Win $50 | 100/100000 | $50 | .05 |
| Win $10 | 500/100000 | $10 | .05 |
| Win $2 | 1000/100000 | $2 | .02 |
| Lose | 1- all the above | 0 |  |

 The sum of the last column is only about $.345, so on average the mathematical expectation is $.345 − 1 = −$.655 each time you play. This means you should expect to lose about 66¢ on average per game played.

Adding a “get a free card” card increases your probability, so it’s increasing the mathematical expectation.