Solutions to Vector Worksheet - Distance Between a Point and a Line/Plane

1. <1, 2, -2>
2. <1/3, 2/3, -2/3>
3. <8/3, 16/2, -16/3>
4. <2, -1, 0> or <0, 1, 1> or anything that produces a dot product of 0.
5. SP = (12 - 52)/5 = -40/5 = -8
6. $\vec{vp}$ = -8/5·<3, -4, 0> = <-24/5, 32/5, 0>
7. $\vec{oc}$ = <4, 13, 5> - <-24/5, 32/5, 0> = <44/5, 33/5, 5>
8. d = $|5\cdot 2 + 12\cdot(-7) - 8|/\surd(12^2 + 5^2) = 82/13$
9. d = $|3\cdot 1 - 4\cdot 5 + 12\cdot 3 - 5|/\surd(3^2 + 4^2 + 12^2) = 19/13$
10. d = $|8\cdot(-6) + 15\cdot 4|/\surd(8^2 + 15^2) = 46/17$
11. The line is 4x - 3y = 26, so d = |4·2 - 3·1 - 26|/5 = 21/5
12. (4, 0, 0) lies on the first plane. So d = $|2\cdot 4 + 5\cdot 0 - 3\cdot 0 - 15|/\surd(2^2 + 5^2 + 3^2) = 7/\surd 34 = 7\surd 34/34$
13. |12x - 5y - 9|/13 = 4 ⇒ |12x - 5y - 9| = 52, so 12x - 5y = 61 or 12x - 5y = -43
14. 7·11 + 24(-18) - 60 = -415, and 7·18 + 24(-20) - 60 = -414, and since they have the same sign, they are on the same half plane.
15. $\vec{AB}$ = <1, -5, -3>, so the plane has equation x - 5y - 3z = (3 - 5(7) - 3(1) =) 35
16. $\vec{AB}$ = <3, 4, -1> and $\vec{AC}$ = <-3, -7, 0>, so $\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & -1 \\ -3 & -7 & 0 \end{vmatrix} = -7\vec{i} + 3\vec{j} - 9\vec{k}$, so the plane is 7x - 3y + 9z = (7·1 - 3·2 + 9·3 =) 28
17. A = $\frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{1}{2} \surd(7^2 + 3^2 + 9^2) = \frac{1}{2} \surd 139$
18. $\vec{BA}$ = <-3, -4, 1> and $\vec{BC}$ = <-6, -11, 1> $\cos\theta = \frac{18 + 44 + 1}{\sqrt{26}\sqrt{158}} \Rightarrow \theta = 10.5997°$
19. Using the vectors from #16, $\cos\theta = \frac{-9 - 28}{\sqrt{26}\sqrt{58}} \Rightarrow h = 162.33°$
20. $\vec{r}$ = <2, 6, 3> + d<6, -7, -7>
21. $\vec{AB}$ = <6, -7, -7> and $\vec{AC}$ = <2, -14, 1>, so $\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -7 & -7 \\ 2 & -14 & 1 \end{vmatrix} = -105\vec{i} - 20\vec{j} - 70\vec{k}$, so the plane is 21x +4y + 14z = (21·2 + 4·6 + 14·3 =) 108
22. The area of the parallelogram formed by $\vec{AB}$ and $\vec{AC} = |\vec{AB} \times \vec{AC}| = 5\surd(21^2 + 4^2 + 14^2) = 5\surd 653$. So h = A/b = $5\surd 653/\surd(6^2 + 7^2 + 7^2) = 5\surd 653/\surd 134 = 5\surd(653/134)$

23. $\overrightarrow{AB}$ = <6, -7, -7> and $\overrightarrow{AD}$= <2, 6, -6>, so $\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -7 & -7 \\ 2 & 6 & -6 \end{vmatrix} = 84\vec{i} + 22\vec{j} - 50\vec{k}$, so

the area of the parallelogram formed by $\overrightarrow{AB}$ and $\overrightarrow{AD}$ = $2\sqrt{(42^2 + 11^2 + 25^2)}$ = $2\sqrt{2510}$. So h = A/b = $2\sqrt{2510}/\sqrt{(6^2 + 7^2 + 7^2)} = 5\sqrt{2510}/\sqrt{134} = 5\sqrt{(1255/67)}$.

24. d = $|21{\cdot}4 + 4{\cdot}12 + 14{\cdot}(-3) - 108)/\sqrt{(21^2 + 4^2 + 14^2)} = 18/\sqrt{653}$

25. Area = $\frac{1}{2}\,|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\, 5\sqrt{653} = 5\sqrt{653}/2$

26. Area = $\frac{1}{2}$ b·h = $\frac{1}{2}\,|\overrightarrow{AB}|$·distance from C to $\overrightarrow{AB}$ = $\frac{1}{2}\sqrt{134}\cdot(5\sqrt{(653/134)}) = 5\sqrt{653}/2$

27. $\overrightarrow{BA}$ = <-6, 7, 7> and $\overrightarrow{BC}$ = <-4, -7, 8>, so $\cos\theta = \dfrac{24 - 49 + 56}{\sqrt{134}\sqrt{4^2 + 7^2 + 8^2}}$, so θ = 76.362°.

28. r = <8, -1, -4> + d·<-4, 13, 1>

29. $\overrightarrow{CB}$ = <4, 7, -8> and the area of the parallelogram formed by $\overrightarrow{CB}$ and $\overrightarrow{BD}$ is the magnitude of $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 13 & 1 \\ 4 & 7 & -8 \end{vmatrix} = -111\vec{i} - 28\vec{j} - 80\vec{k}$ which is $\sqrt{19505}$, so h = $\sqrt{19505}/|BD|$ = $\sqrt{19505}/\sqrt{(16+169+1)} = \sqrt{(19505/186)}$

30. We found the area of the parallelogram formed by $\overrightarrow{AB}$ and $\overrightarrow{AD}$ in problem # 23. It is $2\sqrt{2510}$. So now h = A/b = $2\sqrt{2510}/\sqrt{(16 + 169 + 1)} = 5\sqrt{2510}/\sqrt{186} = 5\sqrt{(1255/93)}$.

31. 6x - 7y - 7z = (6·4 - 7·(-8) - 7·4 =) 52

32. 6x - 7y - 7z = (6·4 - 7·(12) - 7·(-3) =) -39

33. (-3, -10, 0) lies on the first plane (and do ∞ more points) so d = $|6(-3)-7(-10)+7{\cdot}0 + 39|/\sqrt{186} = 91/\sqrt{186}$

34. They both contain point B, so d = 0.

35. r = <8, -1, -4> + d·<-4, -7, 8>

36. r = <4, 12, -3> + d·<-4, -7, 8>

37. Find the distance between (4, 12, -3) and line $\overleftrightarrow{BC}$. $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 13 & 1 \\ -4 & -7 & 8 \end{vmatrix} = 111\vec{i} + 28\vec{j} + 80\vec{k}$. So the distance is $\sqrt{19505}$,/$\sqrt{(16+49+64)} = \sqrt{(19505/129)}$

38. r = <2, 6, 3> + d<2, 6, 6>

39. Find a vector ⊥to both lines: n = $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 6 & 6 \\ -4 & -7 & 8 \end{vmatrix} = 90\vec{i} - 40\vec{j} + 10\vec{k}$ Then find a vector connecting the lines: c = AB . So the scalar projection of AB on n = $75\sqrt{2}/14$