Accelerated Precalculus Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

DeMoivre’s Theorem Period\_\_\_\_\_Date\_\_\_\_\_\_\_\_\_\_

Let’s recap yesterday’s discoveries:

1. Write a = -4 + 3***i*** in polar form.

2. Write b = 5 + 12***i*** in trigonometric form.

3. Use the polar forms of these numbers to find a·b and b/a in polar form.

4. Convert the expressions for a·b and b/a above to standard form.

5. Check the answers to #4 by performing the appropriate operations on a and b in standard form.

6. Write a3 in polar form.

Now, consider the following problem: x3 = 44 + 117***i*** . Since this is a cubic function, there should be 3 complex roots. How can we find all 3?

First, consider this complex number written in polar form: x3 = 125(cos 69.39˚ + ***i***·sin 69.39˚) (Remember how we would find that angle? ) How could we find the distance from x to the origin? (Remember that this is also called the absolute value of x, but can also be called the modulus, magnitude, or, as many of you probably see it, the radius.)

Hopefully, you recognize that since we cubed the absolute value of a to get the absolute value of a3, we must take the cube root of the magnitude of x3 to get the magnitude of x, so |x| = 5.

Next we need to find the measure of the angle. Since we multiplied the original angle by 3 to get x3, let’s divide this angle by 3 to find the angle made by x. 69.39˚/3 = 23.13˚, so one root should be 5(cos 23.13˚ + ***i***·sin 23.13˚) = 4.598 + 1.964***i***.

As you notice, that is not the number that we cubed, so let’s look at a way to find the other two roots. As you might have figured out, when we got 69.39˚ for our angle, that is not the only possible angle, so let’s look for something coterminal! 69.39˚ + 360˚ = 429.39˚ has the same sine and cosine values as 69.39˚, so we can consider the fact that

x3 could also equal 125(cos 429.39˚ + ***i***·sin 429.39˚). When we divide this angle by 3, we get 143.13˚, so another value of x is 5(cos 143.13˚ + ***i***·sin 143.13˚) = -4 + 3 ***i***. The third possibility for the value of x can be found if we let x3 = 125(cos 789.39˚ + ***i*** ·sin 789.39˚). Then x = 5(cos 263.13˚ + ***i*** ·sin 263.13˚) = -.5891 – 4.964 ***i***.

If you look at the polar form of all 3 of these roots, what is the modulus of all three possible values of x?

What is the relationship between the angles of these three roots? (Graph them if you don’t see this.)

Don’t you just love the symmetry?

Let’s work a simpler problem. Find all the fourth roots of 81. This means that x4 = 81.

What is the radius of all these roots?

What’s the relationship between the angles?

Try to write a theorem about finding all n of the nth roots of any complex number.

Homework:

1. Find exact values of all of the cube roots of 8 ***i***.

2. Find exact values of all of the square roots of -49 ***i***.

3. Find approximate values of all fourth roots of 3 + 4 ***i***.

4. Find approximate values of all 5 roots of 32.

5. If 3(cos 48˚ + ***i*** ·sin 48˚) is one of the cube roots of k, a) find the two other roots in polar form, b) sketch all the roots, and c) write k in standard form.

6. If 3 – 5i is one of the square roots of p, a) write the other root, b) sketch both roots and c) write p in standard form.

7. If 2.3(cos 322˚ + ***i*** ·sin 322˚) is one of the fourth roots of x, a) find the other 3 roots in polar form, b) write all four roots in standard form, c) sketch all four roots and d) write x in polar and in standard form.

DeMoivre’s Theorem Answers:

1. Find exact values of all of the cube roots of 8***i***.

 x3 = 8***i*** = 8(c***i***s 90º) so x = 2(c***i***s 30º) or 2(c***i***s 150º) or 2(c***i***s 270º) - meaning

 x = + ***i*** or x = -+ ***i*** or x = -2***i***.

2. Find exact values of all of the square roots of -49 ***i***.

 x2 = -49 ***i*** = 49(c***i***s 270º), so x = 7(c***i***s 135º) or x = 7(c***i***s 315º) - meaning

 x = -3.5 + 3.5***i*** or x = 3.5 - 3.5***i***

3. Find approximate values of all fourth roots of 3 + 4***i***.

 x4 = 3 + 4***i*** = 5(c***i***s 53.13º), so x = 1.495(c***i***s 13.28º) or x = 1.495(c***i***s 103.28º) or

 x = 1.495(c***i***s 193.28º) or x = 1.495(c***i***s 283.28º) – meaning

 x = 1.455 + .3436***i*** or x = -.3436 + 1.455***i*** or x = -1.455 - .3436***i*** or x = .3436 - 1.455***i***

4. Find approximate values of all 5 roots of 32.

 x5 = 32 = 32(c***i***s 0º), so x = 2(c***i***s 0º) or x = 2(c***i***s 72º) or x = 2(c***i***s 144º) or

 x = 2(c***i***s 216º) or x = 2(c***i***s 288º) – meaning

x = 2 or x = .6180 + 1.902***i*** or x = -1.618 + 1.176***i*** or x = .6180 - 1.902***i***

or x = -1.618 - 1.176***i***

5. If 3(cos 48˚ + ***i*** ·sin 48˚) is one of the cube roots of k, a) find the two other roots in polar form, b) sketch all the roots, and c) write k in standard form.

 a) The other two roots are 3(cos 168º + ***i***sin 168º) and 3(cos 288º + ***i***sin 288º)

 c) so k ≈ -21.84 + 15.89***i***

6. If 3 – 5i is one of the square roots of p, a) write the other root, b) sketch both roots and c) write p in standard form.

a) The other root is -3 + 5i so c) p = -16 - 30***i***

7. If 2.3(cos 322˚ + ***i*** ·sin 322˚) is one of the fourth roots of x, a) find the other 3 roots in polar form, b) write all four roots in standard form, c) sketch all four roots and d) write x in polar and in standard form.

a) The other 4 roots are 2.3(c***i***s 52º), 2.3(c***i***s 142º), and 2.3(c***i***s 232)

b) 1.812 – 1.416***i*** , 1.416 + 1.812***i*** , -1.812 + 1.416***i*** , and -1.416 - 1.812***i***

c) x = 27.9841(cos 208º + ***i***sin 208º) or -24.71 – 13.14***i***