Accelerated Precalculus

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3RD

Trig Identities

Unit 6

I. Even and Odd Identities

even

$\cos(-\theta) = \cos\theta$

$\sec(-\theta) = \sec\theta$

odd

$\sin(-\theta) = -\sin\theta$

$\csc(-\theta) = -\csc\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

II. Reciprocal Identities

$\cos\theta = \frac{1}{\sec\theta}$ $\quad \sin\theta = \frac{1}{\csc\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \csc\theta = \frac{1}{\sin\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

III. Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot = \frac{\cos\theta}{\sin\theta}$

IV. Pythagorean Identities

$\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

V. Linear Combinations

$a\cos\theta + b\sin\theta = \sqrt{a^2+b^2}\cos(\theta - k)$

where (a, b) lies on K

VI. Cofunction Identities

$*90° = \pi/2$

$\cos\theta = \sin(90° - \theta)$ $\quad \sec\theta = \csc(90° - \theta)$ $\quad \tan\theta = \cot(90° - \theta)$

$\sin\theta = \cos(90° - \theta)$ $\quad \csc\theta = \sec(90° - \theta)$ $\quad \cot\theta = \tan(90° - \theta)$

VII. Composite Identities

$$\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$$

$$\cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$$

$$\sin(a+b) = \sin a \cdot \cos b + \cos a \cdot \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

VIII. Sum to Product Identities

$$\cos(A+B) + \cos(A-B) = 2\cos A \cos B$$

$$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$$

$$\sin(A+B) + \sin(A-B) = 2\sin A \cos B$$

$$\sin(A+B) - \sin(A-B) = 2\cos A \sin B$$

IX. Double and Half Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \quad \underline{OR} \quad 1 - 2\sin^2\theta \quad \underline{OR} \quad 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

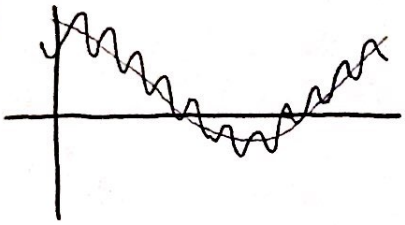
$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} \quad OR \quad \frac{\sin\theta}{1+\cos\theta}$$

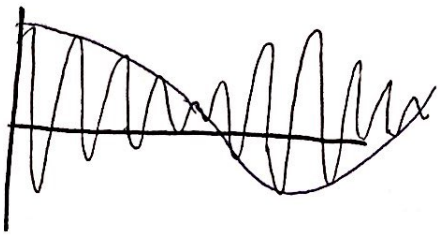
X. Harmonic Analysis (Graphing)

graphs added:



- draw new graph through the middle of the existing graph

graphs multiplied



- draw new graph enveloping the existing graph
- find ONE amplitude

- find amplitude of BOTH graphs
- calculate period of graph by counting how many times it repeats in 360° or 2π

$$B = \frac{2\pi}{\text{period}}$$

XI. Rotating a Point

Rotating a point through an angle θ

$$(x, y) \rightarrow (x\cos\theta - y\sin\theta,\ y\cos\theta + x\sin\theta)$$

transformation matrix:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

*Remember 30-60-90 triangles & 45-45-90 triangles as well as unit circle values