Acc Precalc

Polar Equations and Graphing Study Guide

Unit 10

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3RD

I. Plotting Polar Points

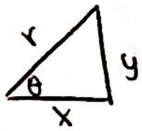
→ Ordered pair (r, θ)

- r = radius from origin
- θ = standard position angle

→ negative θ → graph as you would a unit circle (still SPA)

→ negative r → graph a negative radius (moving in the opposite direction as given)

II. Polar and Cartesian Points



$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ r &= \sqrt{x^2+y^2} \\ \theta &= \tan^{-1}\left(\frac{y}{x}\right) \end{aligned}}$$

* double check quadrant

- polar to cartesian

$(r, \theta) \Rightarrow (r\cos\theta, r\sin\theta)$

- cartesian to polar

$(x, y) \Rightarrow (r, \theta)$

* graph/sketch to visualize

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}(y/x)$

III. Polar and Cartesian Equations

- cartesian to polar (x & y to r & θ)

1. Replace
   $x = r\cos\theta$
   $y = r\sin\theta$

2. Solve for R

• polar to cartesian equation (r & θ to x & y)

① clear fractions

② Replace

$$r = \sqrt{x^2+y^2}$$

$$r^2 = x^2+y^2$$

$$x = r\cos\theta \quad \text{or} \quad \cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

$$y = r\sin\theta \quad \text{or} \quad \sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

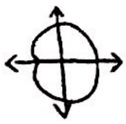
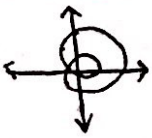
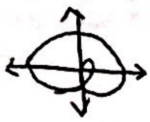
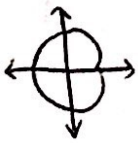
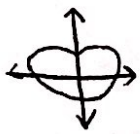
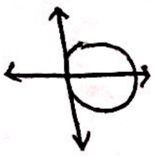
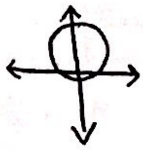
$$\tan\theta = \frac{y}{x}$$

• complete the square $\left(\frac{b}{2}\right)^2$

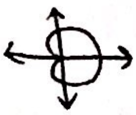
③ Write in some simplified form

Ⓘⓥ Special Polar Graphs

[A] Limaçons

	cosine		sine	
	$r = a + b\cos\theta$		$r = a + b\sin\theta$	
	• symmetric w/ x axis		• symmetric w/ y axis	
$a > b$		- no loop - sometimes indentation		• no loop • indentation
$a < b$		• loop • negative radius at some point		• loop • neg radius at a point
$b < 0$ (negative)		• reflected over y axis		• reflected over x axis
$a = 0$		circle		circle

[B] Cardiods

cosine	sine
$r = a + a\cos\theta$	$r = a + a\sin\theta$
• symmetric to x axis	• symmetric to y axis
	

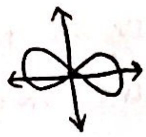
[C] Lemniscates

"infinity"

cosine

$r^2 = a^2 \cos 2\theta$

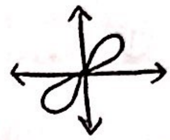
- symmetric w/ x axis



sine

$r^2 = a^2 \sin 2\theta$

- symmetric w $y = x$ or $\theta = \pi/4$



[D] Roses

cosine

$r = a \cdot \cos(b\theta)$

- always has a petal on the positive x axis

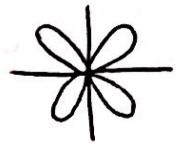
sine

$r = a \cdot \sin(b\theta)$

- no petals lie on the x axis

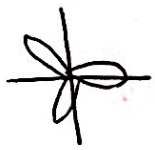
b is even

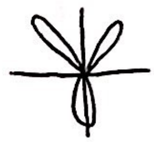
\# of petals = $2b$



b is odd

\# of petals = b





- a affects length of petals

$90°/b$ = angle of a max point on a petal

$\frac{360°}{b}$ = angle between petal "tips"

* when b is a fraction (p/q)

→ complete graph = $2\pi q$

→ π/p = # of radians in rose petal

[E] Lines

$Ax + By = C$ is $r = \dfrac{c}{a\cos\theta + b\sin\theta}$

slope : $-\frac{a}{b}$

x int : $\frac{c}{a}$

y int : $\frac{c}{b}$

V. Intersections and simultaneous Solutions

1. set equations equal to each other
2. solve for θ (can be multiple)
3. plug in all θ values into original equation(s) & solve for corresponding R value
4. write solutions in polar form (r, θ)

★note: number of points of intersection may not always be the same as number of simultaneous solutions

* important to remember trig identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

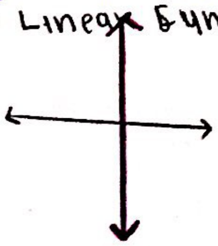
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$1 - 2\sin^2\theta$$
$$2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

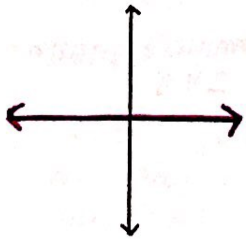
★ and more...

VI Symmetry and max Radius

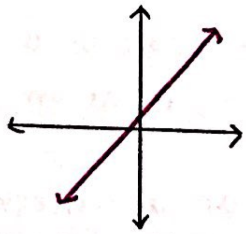
- Linear Symmetry



$\theta = \frac{\pi}{2}$



$\theta = 0$ or "pole"



$\theta = \pi/4$

- Rotational Symmetry

→ max r values

- occur when cosine and sine values are -1 or 1
  - disregard "unimportant" info in equation

VII Conics

$$r = \frac{ep}{1 \pm e\cos\theta} \qquad r = \frac{ep}{1 \pm e\sin\theta}$$

e = eccentricity (c/a)

$e > 1$ hyperbola
$e = 1$ parabola
$e < 1$ ellipse

$|p|$ = distance between focus (pole) and directrix

hyperbola: $a^2 + b^2 = c^2$

ellipse: $a^2 = b^2 + c^2$