Accelerated Precalculus

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3rd

Parametric Equations Study Guide
Unit 9

I. Intro to Parametric Equations

- parametric equation: two or more equations written as a function of one or more variables called parameters (T)

- when graphing always check:
  - t min / t max
  - t step
  - x min / x max
  - y min / y max

II. Writing functions in parametric form

- choose an expression for x involving the parameter (T)
- then write y substituting the expression involving (T) for x

III. Writing parametric equations given two points

⇒ (x_1, y_1) and (x_2, y_2)

$$\boxed{\begin{aligned} x &= x_1 + (x_2 - x_1)t \\ y &= y_1 + (y_2 - y_1)t \end{aligned}}$$

⇒ a line with an x int (a) and y int (b)

(a, 0) and (0, b)

$$\boxed{\begin{aligned} x &= at \\ b &= b + (-b)t \end{aligned}}$$

IV. Finding points of intersection

- use distance formula

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

⇒ but substitute parametric equations x_1, y_1 and x_2, y_2 into eq.
↳ this will create a FUNCTION of distance & time

- use $\boxed{\text{trace}}$ on calculator to find how far apart 2 eq. are at any given time

minimum of a d vs t graph	
x value	= time where they meet / get closest
y value	= distance apart at closest point

- if the points do intersect, the function should touch the x-axis
- find closest meeting point by finding <u>minimum</u> using $\boxed{\text{2ND}}\boxed{\text{TRACE}}$

(V) Eliminating Parameters

- solve algebraically to eliminate [T] to result in an equation with x & y terms only
  - elimination
  - substitution

* you can also use trig identities & conics to eliminate the parameter

* when t is the exponent, try squaring or multiplying the x & y terms to successfully eliminate the parameter

* you can always <u>CHECK</u> by graphing

<u>IMPORTANT TRIG IDENTITIES</u>:

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta \quad \text{OR} \quad \sec^2\theta - \tan^2\theta = 1$$

***be sure to memorize other trig identities in case they are needed to eliminate the parameter**

(VI). Without eliminating the parameter....

find...

[A] slope $\boxed{\dfrac{\Delta y}{\Delta x}}$ OR $\dfrac{(y_2 - y_1)}{(x_2 - x_1)}$

[B] x intercept
- set $y = 0$
- solve for t
- plug t into x= equation

$(x, 0)$

[C] y intercept
- set $x = 0$
- solve for t
- plug t into y= equation $(0, y)$

[D] points of intersection to a graph

- simply substitute the x and y values of the parametric equations into the given equation
- set new eq. equal to 0
- solve for t (s)
- plug in t value(s) into original parametric equation to find x & y coordinates of the point(s) of intersection

VII Inverses of Parametric Equations

- simply switch x and y equations :)

VIII Transformations

- pretty simple → just add / subtract / multiply / divide transformations to their appropriate x or y equations

x- equation: left / right / period / phase shift

y-equation: up / down / amplitude / vertical shift

IX Ferris Wheels

- must know:
  - size (radius / diameter)
  - speed (period)
  - direction (cw / ccw)
  - initial point
- create __TWO__ equations to model the movement of the ferris wheel

  ① x =
  models the right / left displacement as a function of time

  ② y =
  models the height as a function of time
- use these two equations to write a parametric equation modeling the movement of the ferris wheel

  ↳ use $\boxed{\text{TRACE}}$ or $\boxed{\text{TABLE}}$ to find position at any given time (& vice versa)

(X) Conics

- conics can oftentimes be modeled by parametric equations

(1) Circles

$(x-h)^2 + (y-k)^2 = r^2$

$$\boxed{\begin{aligned} x &= h + r\cos t \\ y &= k + r\sin t \end{aligned}}$$

- vertex (h,k)
- radius $= r$

(2) parabolas

$(x-h)^2 = \pm 4p(y-k)$ opens up/down ∪ OR ∩

$$\boxed{\begin{aligned} x &= t \\ y &= k + a(t-h)^2 \end{aligned}}$$

- vertex (h,k)
- dilation factor a

$(y-k)^2 = \pm 4p(x-h)$ opens left/right ⊂ OR ⊃

$$\boxed{\begin{aligned} x &= h + a(t-k)^2 \\ y &= t \end{aligned}}$$

(3) ellipses

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{OR} \quad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\boxed{\begin{aligned} x &= a\cos t + h \\ y &= b\sin t + k \end{aligned}}$$

center at (h,k)
dilations of a & b
$b^2 + c^2 = a^2$

$$\boxed{\begin{aligned} x &= b\cos t + h \\ y &= a\sin t + k \end{aligned}}$$

(4) hyperbolas

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad \text{OR} \quad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\boxed{\begin{aligned} x &= a\sec t + h \\ y &= b\tan t + k \end{aligned}}$$

center at (h,k)
x & y dilations of a & b
asymptotes: $\frac{\Delta y}{\Delta x}$

$$\boxed{\begin{aligned} x &= b\tan t + h \\ y &= a\sec t + k \end{aligned}}$$

XI. Rotating a Parametric Equation

→ to rotate any parametric equation θ degrees counterclockwise...

$$x_2(t) = x_1 \cos\theta - y_1 \sin\theta$$
$$y_2(t) = x_1 \sin\theta + y_1 \cos\theta$$

XII Projectile Motion

to model projectile motion, use equations:

$$x = v_0 \cos\theta \, t \pm \text{left/right position}$$
$$y = v_0 \sin\theta \, t - (g)t^2 \pm h_0$$

where: v_0 = initial velocity

θ = angle of projection

g = gravity

↳ either -16 ft/sec^2

or -4.9 m/sec^2

h_0 = initial height

* make sure to convert mph to ft/sec

$$\frac{x \ \cancel{mi}}{1 \ \cancel{hr}} \cdot \frac{5280 \ ft}{1 \ \cancel{mi}} \cdot \frac{1 \ \cancel{hr}}{3600 \ sec} = \_\_\_\_ \text{ ft/sec}$$

1. maximum height

1) complete the square

2) use $-\frac{b}{2a}$ to find the time at which the object passes through its max height

↳ then plug in (t) to the y equation & solve for max height

2. distance travelled

- object returns to the ground when its height (y) is 0

↳ so set y = 0 and solve for t (use quadratic formula)

↳ then plug t into x equation & solve for distance

3. time in flight

• set y equation equal to zero, & use quadratics to solve for t