Acc Precalculus

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9-19-17
3RD

Conics - Study Guide
Unit 3

I. Circles

→ general form $\boxed{x^2 + y^2 + Ax + By + C = 0}$

→ standard form $\boxed{(x-h)^2 + (y-k)^2 = r^2}$

center : (h, k)
radius : r

- when completing the square (to get from general form to standard form)
  - make sure coefficient = 1
  - $\left(\frac{b}{2}\right)^2$ = c term

II. Parabolas

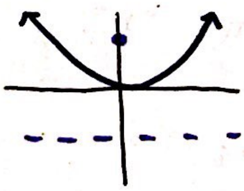
parabola: the set of points equidistant from a fixed point (the focus) and a fixed straight line (the directrix)

p = focal distance/length

① $\boxed{(x-h)^2 = \pm 4p(y-k)}$

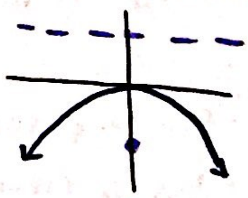
x term is squared ⇒ opens up/down

vertex (h, k)
focus $(h, k \pm p)$
directrix $y = k \pm p$



+p

OR

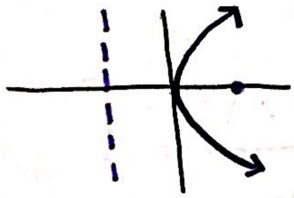


−p

② $\boxed{(y-k)^2 = \pm 4p(x-h)}$

y term is squared ⇒ opens left/right

vertex (h, k)
focus $(h \pm p, k)$
directrix $x = h \pm p$



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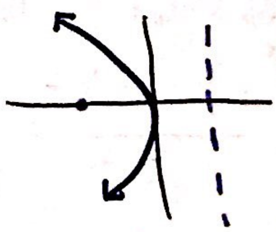

−p

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more forms for parabolas:

→ Standard form $y = ax^2 + bx + c$

→ vertex form $y = a(x-h)^2 + k$

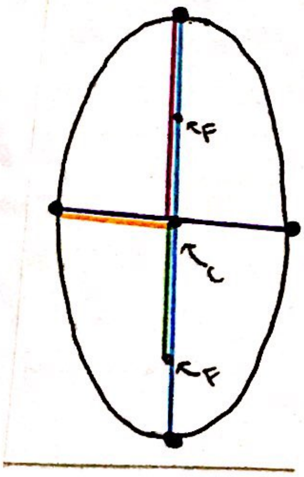
→ factored (intercept) form $y = a(x - r_1)(x - r_2)$

• if (x,y) is a point on a parabola, then distance to focus = distance to directrix

III. Ellipses

ellipsis: the set of points so that the sum of their distances from two fixed points is constant

→ parts of an ellipse



→ major axis

→ semi-major axis (a)

→ vertices

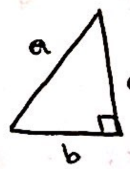
→ focal distance (c)

→ minor axis

→ Semiminor axis (b)

→ Covertices

⇒



$\boxed{b^2 + c^2 = a^2}$

→ eccentricity from any point "p" ⇒ $\boxed{\frac{pf \text{ (focus)}}{pd \text{ (directrix)}} \text{ OR } \frac{c}{a}}$

- for a parabola $e = 1$
- for a circle $e = 0$
- for an ellipse $e = \frac{c}{a}$ $e < 1$

- for a hyperbola $e > 1$

↳ smaller eccentricity = more circular (foci = closer)
larger eccentricity = more oval-like (foci = farther)

→ equation of an ellipse

$$\boxed{\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1}$$

where a^2 is under the larger term (term w/ major axis)

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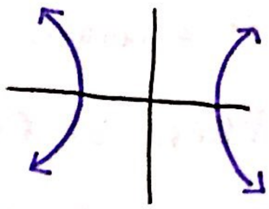
IV. Hyperbolas

set of points where the difference of two points is constant

① $\boxed{\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1}$

x^2 term is first

$a^2+b^2=c^2$



center (h,k)

vertices $(h \pm a, k)$

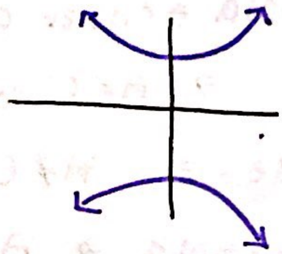
foci $(h \pm c, k)$

asymptotes $y-k = \pm \dfrac{\Delta y}{\Delta x}(x-h)$

② $\boxed{\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1}$

y^2 term first

$a^2+b^2=c^2$



center (h,k)

vertices $(h, k \pm a)$

foci $(h, k \pm c)$

asymptotes $y-k = \pm \dfrac{\Delta y}{\Delta x}(x-h)$

* a^2 will always be under the POSITIVE term

- hyperbolas approach the asymptotes
  - ↳ asymptotes should be written in point slope form

$\boxed{y-k = \pm m(x-h)}$ where $m = \dfrac{\Delta y}{\Delta x}$

→ conjugate axis : joins covertices

→ transverse axis : joins vertices

a= center to vertex
c= center to focus
d= center to directrix

$\Rightarrow e = \dfrac{PF}{PD} = \dfrac{c-a}{a-d} = \boxed{\dfrac{c}{a}}$

$\boxed{d = \dfrac{a^2}{c}}$ d= distance to directrix

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V. Degenerate Conics

the cutting plane passes through the base of the cone

- can obtain a point, a line, or a pair of intersecting lines

↳ difference of 2 squares = 0

VI. General Form of Conics

$$Ax^2 + \underbrace{Bxy}_{\text{rotates conic}} + Cy^2 + Dx + Ey + F = 0$$

so in...

$$\boxed{Ax^2 + Cy^2 + Dx + Ey + F = 0}$$

- parabola ⇒ either a or c = 0 (but not both)
- circle ⇒ A = C but (A or C $\neq 0$)
- ellipse ⇒ $A \neq C$ and A or C $\neq 0$ and $A * C > 0$
- hyperbola ⇒ $AC < 0$
  ↳ one is pos + one is neg.

Rule $b^2 - 4ac$ is always the same value even when rotated

- parabola $b^2 - 4ac = 0$
- circle $b = 0$
- ellipse $b^2 - 4ac < 0$
- hyperbola $b^2 - 4ac > 0$

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VII. Solving Systems of Equations

↳ Intersections of conics

- use elimination or substitution to solve for one variable
- then plug in to find other variable
- test points by plugging them back into the equations

VIII Converting to y= form

→ expand form (if in standard) to become this order

$$\underbrace{y^2}_{A} + \underbrace{y}_{B} + \underbrace{x^2 + x + \text{constant}}_{C} = 0$$

→ then use quadratic formula to solve for y

$$\boxed{y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

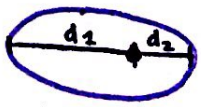
IX. Applications

→ parabolic arch 

- center on y-axis, and put bottom on x-axis
- if given height and width, write equation in standard form

$$(x-h)^2 = \pm 4p(y-k)$$

- use given point (usually endpoints) to write equation
- solve for what problem is asking for

→ elliptical orbits with one object at focus 

- given object at a focus, closest distance (d_2) and farthest distance (d_1)
- $2a = d_1 + d_2$ so $a = \frac{d_1 + d_2}{2}$
- $c = a - d_2$
- $a^2 = b^2 + c^2$
- solve for what problem is asking for

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→ conics with light

ellipse - light rays travel from one focus, bounce off the ellipse, and travel to the other focus

good for spotlights

parabola light rays travel from the focus, bounce off the parabola, and travel parallel to the major axis & perpendicular to the directrix

X. Directrices

ellipses, hyperbolas, and parabolas all have directrices / a directrix

• ellipses & hyperbolas

2 directrices

$$d = \frac{a^2}{c}$$

where d = distance from center / vertex (for parabolas) to the directrix

• parabolas

1 directrix

$$d = p$$

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