

Beginning Trig Study Guide

Unit 4

I. The Unit Circle

① standard position angle

vertex at origin, initial side on positive x-axis, terminal side measured CCW from initial side

② unit circle

circle with a radius of 1

$$\cos \theta = x \text{ value}$$

$$\sin \theta = y \text{ value}$$

→ equation:

$$(\cos \theta)^2 + (\sin \theta)^2 = 1$$

$$x^2 + y^2 = 1$$

③ trig functions

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} \text{ OR } \frac{\sin}{\cos}$$

$$\csc \theta = \frac{1}{\sin \theta} \text{ OR } \frac{\text{hyp}}{\text{opp}}$$

$$\sec \theta = \frac{1}{\cos \theta} \text{ OR } \frac{\text{hyp}}{\text{adj}}$$

$$\cot \theta = \frac{1}{\tan \theta} \text{ OR } \frac{\cos}{\sin} \text{ OR } \frac{\text{adj}}{\text{opp}}$$

③

Q I

$$0^\circ < \theta < 90^\circ$$

$$0 < \theta < \frac{\pi}{2}$$

$$\cos \theta = \text{pos } +$$

$$\sin \theta = \text{pos } +$$

Q II

$$90^\circ < \theta < 180^\circ$$

$$\frac{\pi}{2} < \theta < \pi$$

$$\cos \theta = \text{neg } -$$

$$\sin \theta = \text{pos } +$$

degrees w same sine values are supplementary + are reflected over the y axis (their cosine values are opposite signs)

④

Q_{III}

$$180^\circ < \theta < 270^\circ$$

$$\cos \theta = \text{neg} -$$

$$\pi < \theta < \frac{3\pi}{2}$$

$$\sin \theta = \text{neg} -$$

Q_{IV}

$$270^\circ < \theta < 360^\circ$$

$$\cos \theta = \text{pos} +$$

$$\frac{3\pi}{2} < \theta < 2\pi$$

$$\sin \theta = \text{neg} -$$

degrees w same cosine values add to 360° (2π) and are reflected over the x axis (their sine values have opposite signs)

on the unit circle...

$$-1 \leq \sin \theta \text{ and } \cos \theta \leq 1$$

↳ so $\sec \theta$ and $\csc \theta$ can never be between -1 and 1

⑤ coterminal angles

angles that share a terminal side

$$\theta \pm 360^\circ \quad \text{or} \quad \theta \pm 2\pi$$

⑥

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \Rightarrow \text{equal to the slope}$$

⑦ Reference angles - acute angles formed by dropping an altitude to the x axis

Q_I

$$\theta = \angle r$$

Q_{II}

$$180^\circ - \theta = \angle r$$

Q_{III}

$$\theta - 180^\circ = \angle r$$

OR

$$\theta - \angle r = 180^\circ$$

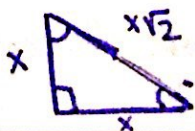
Q_{IV}

$$360^\circ - \theta = \angle r$$

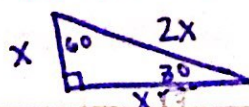
$$\sin(B) = \cos(90 - B)$$

special triangles

45-45-90



30-60-90



⑧ If given 1 trig function, use pythagorean theorem to find missing side and then find other trig functions

⑨ $1 \text{ Radian} = \frac{\text{arc length}}{\text{radius}}$

- Radians have no units

- $360^\circ = 2\pi \text{ radians} = 1 \text{ circle}$

- 1 radian = a little less than 60°

degrees to radians

$$\theta^\circ * \frac{\pi}{180} = \text{Radians}$$

Radians to degrees

$$\theta \text{ Radians} * \frac{180}{\pi} = \text{degrees}$$

* be able to find values of trig functions given a unit circle on graph paper & a protractor

II. Linear and Angular Velocity

→ angular velocity (ω) # of degrees or radians per unit of time $(\frac{\theta}{t})$

ex) two objects connected by an axle/straight line have the same angular velocity

→ linear velocity (v) distance per unit of time $(\frac{d}{t})$

ex) two objects connected by their edges have the same linear velocity

* use dimensional analysis to convert *

- 1 Revolution = $2\pi \text{ radians} = 360^\circ = 1 \text{ circumference}$

- 1 radian = 1 radius

III. Degrees, Minutes, Seconds (DMS)

→ degree ° 360 degrees in 1 circle

→ minute ' $\frac{1}{60}$ degree

→ second " $\frac{1}{3600}$ degree

CALCULATOR
2ND → apps
ANGLE

1. °
2. '
4. > DMS

IV. Trig with a calculator

* UNITS MATTER *

- if problem is in degrees, calculator must be in degrees

- if problem is in radians, calculator must be in radians

- if given a trig function value and you are trying to solve for the angle, use the INVERSE ($^{-1}$)

- temporarily convert to radians in calculator:

2ND → Apps (angle) → 3 (r)

* use double parentheses

V. More trig values

$$\sin(\pi - \theta) = + \sin \theta$$

$$\cos(\pi - \theta) = - \cos \theta$$

$$\tan(\pi - \theta) = - \tan \theta$$

$$\csc(\pi - \theta) = + \csc \theta$$

$$\sec(\pi - \theta) = - \sec \theta$$

$$\cot(\pi - \theta) = - \cot \theta$$

$$\sin(\theta + \pi) = - \sin \theta$$

$$\cos(\theta + \pi) = - \cos \theta$$

$$\tan(\theta + \pi) = + \tan \theta$$

$$\csc(\theta + \pi) = - \csc \theta$$

$$\sec(\theta + \pi) = - \sec \theta$$

$$\cot(\theta + \pi) = + \cot \theta$$

$$\sin(90 - \theta) = \cos \theta$$

$$\cos(90 - \theta) = \sin \theta$$

$$\tan(90 - \theta) = \cot \theta$$

$$\csc(90 - \theta) = \sec \theta$$

$$\sec(90 - \theta) = \csc \theta$$

$$\cot(90 - \theta) = \tan \theta$$

$$\sin(-\theta) = - \sin \theta$$

$$\cos(-\theta) = + \cos \theta$$

$$\tan(-\theta) = - \tan \theta$$

$$\csc(-\theta) = - \csc \theta$$

$$\sec(\theta) = + \sec \theta$$

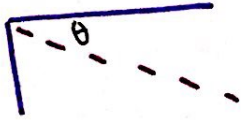
$$\cot(-\theta) = - \cot \theta$$

VI. Applications

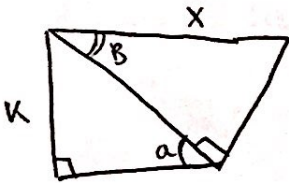
→ angle of elevation



→ angle of depression



→ solving in terms of x



$$\sin(a) = \frac{k}{h}$$

$$\cos(B) = \frac{h}{x}$$

$$h = \frac{k}{\sin a}$$

$$\cos(B) = \frac{k}{\sin a} \div x$$

$$x \cos(B) = \frac{k}{\sin a}$$

$$x = \frac{k}{\sin(a) \cos(B)}$$