Accelerated Precalculus

Emily Thom
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3RD

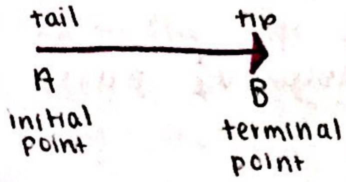
2D Vectors

Unit 8

I. Intro to 2D Vectors

- vectors have magnitude and direction

$\overrightarrow{AB}$



$|\overrightarrow{AB}|$ = magnitude or length

- position vectors start at the origin
- unit vectors are vectors 1 unit long

$\vec{i}$ 1 unit in x direction

$\vec{j}$ 1 unit in y direction

] → component form

- add vectors "tip" to tail

→ **notation of vectors**

$3\vec{i} + 4\vec{j}$ (unit vector)

$\langle 3, 4 \rangle$

II. Dot Products

- magnitude of a vector

$\sqrt{x^2+y^2}$

- dot product

$$\boxed{\langle r, s \rangle \cdot \langle t, u \rangle = rt + su}$$

- cosine functions have signs corresponding to the dot product at the angles

$$\boxed{\vec{u} \cdot \vec{v} = r_1 r_2 (\cos(a-b))}$$

where:

$\vec{u}$ = radius r_1 and $\theta = a$

$\vec{v}$ = radius r_2 and $\theta = b$

- finding the angle between two vectors

$$\boxed{\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}\right)}$$

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- orthogonal (perpendicular) vectors
  - slopes must be opposite reciprocal
  - dot product will be 0
    - ↳ so the angle will be 90°

III.) Vector Projections

- scalar projection
  - the length of a shadow $\vec{v_1}$ casts on $\vec{v_2}$

$$sp = |\vec{v_1}| \cos\theta$$

$$sp = \frac{\vec{v_1} \cdot \vec{v_2}}{|\vec{v_2}|}$$

- vector projection
  - the vector projection of $\vec{v_1}$ onto $\vec{v_2}$ is the vector in the same direction as v_2, whose length is the scalar projection of $\vec{v_1}$ on $\vec{v_2}$

$$vp = sp \text{ of } \vec{v_1} \text{ to } \vec{v_2} * \text{unit vector of } \vec{v_2}$$

- vector component
  - the vector component of $\vec{v_1}$ orthogonal to $\vec{v_2}$ is the vector projection to $\vec{v_2}$ that adds the vector projection of $\vec{v_1}$ on $\vec{v_2}$ to create $\vec{v_1}$

$$OC = v_1 - (vp \text{ of } \vec{v_1} \text{ onto } \vec{v_2})$$

- obtuse angles
  - ↳ they have negative vector & scalar projections
    - the negative sign indicates an opposite direction as a vector

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IV. Applications

- tension problems
  - if an object is not moving ⇒ forces are equal
    - right / left Net forces must be zero
    - up forces must equal down forces (weight)
  - make sure to find the angle and use sine/cosine in the right components
    - ↳ sine doesn't always mean up/down
  - set up a system of equations & solve (matrices)

V. Vector Equations of a Line

$$\boxed{\vec{r} = \vec{OA} + d\,\vec{AB}}$$

- $\vec{OA}$ is a point on the line
- $\vec{AB}$ is the slope of the vector $\left(\frac{\Delta y}{\Delta x}\right)$

  $\langle \Delta x, \Delta y \rangle$

- to find the points x units away from a point on a line, let $d = \pm x$ and make $\vec{AB}$ a unit vector

VI. Distances between a Point and a Line

- point $p = (x_1, y_1)$
- line $\ell = Ax + By = C$

- $\langle A, B \rangle$ is perpendicular to line ℓ
- $(0, \frac{C}{B})$ lies on the line ℓ, so a vector joining p to the line $= \langle x_1, y_1 - \frac{C}{B} \rangle$
- the scalar projection of $\langle x, y_1 - \frac{C}{B} \rangle$ onto $\langle A, B \rangle =$

$$\frac{\langle x_1, y_1 - \frac{C}{B} \rangle \langle A, B \rangle}{|\langle A, B \rangle|}$$

$$\boxed{d = \left| \frac{Ax_1 + By - C}{\sqrt{A^2 + B^2}} \right|}$$

→ the distance between a point (A) to line (ℓ)

↳ the scalar projection of $\vec{u}$ onto $\vec{n}$ where

$\vec{u}$ joins A to any point on the line and

$\vec{n}$ is perpendicular to ℓ

• **perpendicular vectors**

a vector $\perp$ to $Ax + By = C$ is $\langle A, B \rangle$

↳ just take the coefficients to get the perpendicular vector