**Introduction to the Unit Circle**

**An Investigative Task in Parts:**

Part I:

A ***standard position angle*** is an angle with vertex at the origin with its initial side lying on the positive x-axis and whose terminal side is counterclockwise from that initial ray.

Using a protractor, draw a 30° standard position angle below. Then draw a 70° standard position angle, a 90° standard position angle and a 120° standard position angle.

Where will the terminal sides of acute angles in standard form be found?

Where will the terminal sides of obtuse angles in standard form be found?

Where will the terminal side of a right angle in standard form be found?

In some triangles, the sine and cosine values of an angle are actually sides of the triangle. What must be true of all such triangles?

If we draw infinitely many of these triangles with standard position angles, what shape would be formed?

Part II:

A ***unit circle*** is a circle with radius 1 unit. Using your compass, construct a large unit circle on your graph paper with center at the origin. (Remember that your unit can be as large as you wish.) Draw a 40° standard position angle on your unit circle. Explain how to estimate values for cos 40° and sin 40°.

Using your method, find cos 40°\_\_\_\_\_\_\_\_\_\_ and sin 40°\_\_\_\_\_\_\_\_\_\_.

In order to get decimal values for cos 40° and sin 40°, how should you construct your unit circle?

Construct another unit circle using that method, and use it to estimate values for cos 40° and sin 40° again – accurate to the hundredths place. cos 40° = \_\_\_\_\_\_\_\_\_ and

sin 40° = \_\_\_\_\_\_\_\_\_\_.

Use your calculator to check your results.

Use this unit circle and a protractor to estimate these values.

cos 80° = \_\_\_\_\_\_\_\_\_ sin 80° = \_\_\_\_\_\_\_\_\_\_

cos 45° = \_\_\_\_\_\_\_\_\_ sin 45° = \_\_\_\_\_\_\_\_\_\_

sin 70° = \_\_\_\_\_\_\_\_\_ cos 70° = \_\_\_\_\_\_\_\_\_\_

cos 10° = \_\_\_\_\_\_\_\_\_ sin 10° = \_\_\_\_\_\_\_\_\_\_\_

On the unit circle, cos θ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

and sin θ= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What is the equation of the unit circle?

What does that mean in terms of the trig functions?

Part II B: There are actually 6 trig functions:

secant θ or sec θ =  cosecant θ or csc θ =  cotangent θ or cot θ = 

Use this information to estimate the following values:

sec 40° = \_\_\_\_\_\_\_\_\_ csc 40° = \_\_\_\_\_\_\_\_\_\_

sec 80° = \_\_\_\_\_\_\_\_\_ csc 80° = \_\_\_\_\_\_\_\_\_\_

sec 45° = \_\_\_\_\_\_\_\_\_ csc 45° = \_\_\_\_\_\_\_\_\_\_

csc 70° = \_\_\_\_\_\_\_\_\_ sec 70° = \_\_\_\_\_\_\_\_\_\_

sec 10° = \_\_\_\_\_\_\_\_\_ csc 10° = \_\_\_\_\_\_\_\_\_\_\_

Part III:

So far the only trig values that make sense are those in a right triangle. But if we use a unit circle, we can find more trig values.

Find cos 120° using a calculator.\_\_\_\_\_\_\_\_\_\_

Now draw a 120° standard position angle on your unit circle. How can we use the unit circle to find cos 120°?

How could we determine sin 120°?

Check your answer with a calculator.

Use the unit circle and a protractor to find each of the following accurate to the hundredths place.

cos 90° = \_\_\_\_\_\_\_\_\_\_\_\_ sin 90° = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

cos 140° = \_\_\_\_\_\_\_\_\_\_\_ sin 140° = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

cos 135° = \_\_\_\_\_\_\_\_\_\_\_ sin 135° = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

sec 155° = \_\_\_\_\_\_\_\_\_\_\_ csc 130° = \_\_\_\_\_\_\_\_\_\_\_\_

If 0° < θ < 90°, the cosine of θ is \_\_\_\_\_\_\_\_\_\_\_\_\_ and the sine of θ is \_\_\_\_\_\_\_\_\_\_\_ because θ is in quadrant \_\_\_\_\_\_\_\_.

If 90° < θ < 180°, the cosine of θ is \_\_\_\_\_\_\_\_\_\_\_\_ and the sine of θ is \_\_\_\_\_\_\_\_\_\_ because θ is in quadrant \_\_\_\_\_\_\_.

Name another angle with the same sine value as 100°. \_\_\_\_\_\_\_\_\_\_Why are these sine values the same?

What is the relationship between the measures of the angles?

What’s true of the cosines of these two angles?

Find an angle with the same sine value as 27°, 134°, and 108°.

Part IV:

In geometry, angles had to be between 0° and 180°, but not in trigonometry. Draw a 250° standard position angle on the axis below. Estimate the cos 250°\_\_\_\_\_\_ and sin 250°\_\_\_\_\_\_\_\_\_\_using the unit circle concept. Check using a calculator. Now use your unit circle and a protractor to estimate each of the following values.

cos 180° = \_\_\_\_\_\_\_\_\_\_\_ sin 180° = \_\_\_\_\_\_\_\_\_\_\_\_\_

cos 240° = \_\_\_\_\_\_\_\_\_\_\_\_ sin 240° = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

cos 380° = \_\_\_\_\_\_\_\_\_\_\_ sec 380° = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

sin 235° = \_\_\_\_\_\_\_\_\_\_\_ csc 235° = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

cos 315° = \_\_\_\_\_\_\_\_\_\_\_ sec 315° = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

sin 270° = \_\_\_\_\_\_\_\_\_\_\_\_ csc 270° = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

If 180° < θ < 270°, the cosine of θ is \_\_\_\_\_\_\_\_\_\_\_\_\_ and the sine of θ is \_\_\_\_\_\_\_\_\_\_\_ because θ is in quadrant \_\_\_\_\_\_\_\_.

If 270° < θ < 360°, the cosine of θ is \_\_\_\_\_\_\_\_\_\_\_\_ and the sine of θ is \_\_\_\_\_\_\_\_\_\_ because θ is in quadrant \_\_\_\_\_\_\_.

Name another angle with the same cosine value as 100°. \_\_\_\_\_\_\_\_\_\_Why are these cosine values the same?

What is the relationship between the measures of the angles?

What’s true of the sines of these two angles?

Find an angle with the same cosine value as 127°, 234°, and 58°.

Just by estimating, classify each of the following statements as true or false. If a statement is false, explain why it is false.

\_\_\_\_\_\_\_cos 240° = −.5 \_\_\_\_\_\_\_\_ sin 315° = .7

\_\_\_\_\_\_\_cos 144° = .8 \_\_\_\_\_\_\_\_ sin 270° = −1

Since all ordered pairs on the unit circle represent (cos θ, sin θ) for some angle θ, what does that mean for maximum and minimum values for cos θ and sin θ?

What does that mean for values of sec θ and csc θ?

Part V:

If, in trigonometry, angle measures aren’t limited, then angle measures of −120° and 540° are possible. Think about this idea, and then draw unit circles below and draw angles measuring −120° and 540°. Check these with your teacher.

Draw angles measuring 380°, −20°, −600°, and 840° below.

The angle measuring −120° shares a terminal side with many other angles. Name at least 2 of them.

The angle measuring 540° shares a terminal side with many other angles, too. Name at least 2 of them.

Angles that share a terminal side are called ***coterminal***. Write two ways to create an angle coterminal with a given one.

Are 420° and −300° coterminal? Explain how you know?

Part VI:

A

In the right triangle to the right, what is cos B?

c

In the triangle to the right, what is sin B? b

In the triangle to the right, what is tan B? B C

a

What is a relationship between sin B, cos B and tan B?

Now go back to the unit circle, where cos θ = \_\_\_\_\_\_\_\_\_ and sin θ = \_\_\_\_\_\_\_\_\_\_.

What is the value of tan θ?\_\_\_\_\_\_\_\_\_\_\_

Remembering that all of the standard position angles originate at the origin, what is another name for the expression for tan θ?

Using this information and the unit circle and a protractor, approximate each of the following values.

tan 45° = \_\_\_\_\_\_\_\_\_\_ tan 30° = \_\_\_\_\_\_\_\_\_\_\_

tan 120° = \_\_\_\_\_\_\_\_\_\_ cot 200° = \_\_\_\_\_\_\_\_\_\_

tan 270° = \_\_\_\_\_\_\_\_\_\_ cot 326° = \_\_\_\_\_\_\_\_\_\_

What is the tangent of the angle that the line y = 2x makes with the positive x-axis?

What is the tangent of the angle that the line y = -x makes with the positive x-axis?

The tangent of standard position angle θ is ½. Write an equation of the line that contains the terminal ray of θ.

Part VII:

When a standard position angle is formed, it often helps to drop an altitude to the x axis to create a right triangle. In such a triangle, the acute angle with vertex at the origin is called a ***reference angle***. Why do you think we drop the altitude to the x-axis instead of the y-axis?

For each of the quadrants, draw an angle in that quadrant, then drop an altitude to find the reference angle.

I II

III IV

For each of the above figures, describe the relationship between the measures of the actual angle θ and the reference angle ∠r.

Quadrant I Quadrant II Quadrant III Quadrant IV

Find reference angles for each following angles:

The reference angle for 130° is \_\_\_\_\_\_. The reference angle for 340° is \_\_\_\_\_\_.

The reference angle for 230° is \_\_\_\_\_\_. The reference angle for 40° is \_\_\_\_\_\_.

The reference angle for 110° is \_\_\_\_\_\_. The reference angle for −140° is \_\_\_\_\_\_.

The reference angle for 170° is \_\_\_\_\_\_. The reference angle for 740° is \_\_\_\_\_\_.

The reference angle for −230° is \_\_\_\_\_\_. The reference angle for 1340° is \_\_\_\_\_\_.

What is true about the trig functions (sines, cosines, and tangents) of a given angle and of the corresponding reference angle?

So, if we know that tan 36° = .75, name the tangent values of 3 other angles in different quadrants based on that information. (And state the angles, too.)

And if we know that sin 30° is 0.5, name the sine values of 3 other angles in different quadrants based on that information. (And state the angles, too.)

Look again at that triangle in part VI. What is the relationship between the sin B and the cos (90° − B)?

Use that information and the fact that sin 30° = 0.5 to name the **cosine** values of 4 angles on the unit circle.

In geometry you learned about special right triangles. Two of these triangles were known by their angle measures. What are they and what is the relationship between the measures of the lengths of their sides?

Draw a unit circle, and by dropping altitudes to form reference “triangles”, find as many of these special right triangles as you can. (You may use another sheet of paper if you like…)

Now, use your knowledge of these special right triangles to find the exact values for each of the following expressions – without using a calculator.

cos 60° = \_\_\_\_\_\_\_\_\_ tan 240° = \_\_\_\_\_\_\_\_

sin 330° = \_\_\_\_\_\_\_\_\_ cos 135° = \_\_\_\_\_\_\_\_\_

tan 180° = \_\_\_\_\_\_\_\_\_ sin 120° = \_\_\_\_\_\_\_\_

cos 270° = \_\_\_\_\_\_\_\_\_ cot 315° = \_\_\_\_\_\_\_\_

sin 90° = \_\_\_\_\_\_\_\_\_ sec 150° = \_\_\_\_\_\_\_\_\_

tan 30° = \_\_\_\_\_\_\_\_\_ csc 225° = \_\_\_\_\_\_\_\_

sin 210° = \_\_\_\_\_\_\_\_\_ cos 180° = \_\_\_\_\_\_\_\_\_

Part VIII:

Using these reference triangles makes it easy to find one trig function if you’re given another.

For example, if sin θ = 3/5 and if θ terminates in quadrant 2, drawing the reference triangle enables us to see that cos θ = −4/5 and tan θ = −3/4. Explain how you see this.

If cos θ = 5/13 and θ terminates in quadrant IV, find values for the other 5 trig functions of θ.

If the point (8, −15) lies on the terminal side of angle θ, find values for all 6 trig functions of θ.

If tan θ = 5, find values for all other 5 trig functions of θ.

Part IX: THE RADIAN

There are many units used to measure length. Name as many as you can.

There is another way to measure an angle as well. Take any circular object (plastic lids work well for this) and use a tape measure to measure its diameter.

D =

What should be the radius of this object? R =

Use the tape measure to wrap 1 radius length around your object. Look at the central angle subtended (cut off) by this distance. The measure of this central angle is 1 ***radian***. In fact, the number of radians that an angle measures is found by the ratio:

Radian measure = 

There are no units associated with the radian. Why is this?

Estimate the size of 1 radian in degrees.

What is your logic?

What is the circumference of your object? C =

Knowing this, about how many radians are there in a circle?

What is your logic?

Using algebra, exactly how many radians are there in a circle?

Convert each of the following degrees to radians or vice versa:

90° = \_\_\_\_\_\_\_\_\_ radians  radians = \_\_\_\_\_\_\_\_\_\_°

250° = \_\_\_\_\_\_\_\_\_radians  radians = \_\_\_\_\_\_\_\_\_\_°

Explain how change degrees to radians and vice versa.

Radians are often used because they have no units. And everything that you just did in degrees can also be done in radians. Make as many connections as you can with this in mind.