Problems:

Those are the results when we solve 3 equations with 3 unknowns.

Example #1: Example #2: Example #3: Example #4:

2x + y - z = 4 2x + y – z = 4 2x + y – z = 4 2x + y – z = 4

2x + y - z = 6 x + 2x + 4z = 11 x + 2y + 4z = 11 x + 2y + 4z = 11

4x + 2y - 2z = 9 x + y + z = 9 x + y + z = 5 x + y – z = 1

**Example #1:**

A - B → 0 = -2 . I can stop here. Since this yields a statement that is never true – and no values of x, y and z will ever make it true – there is no solution to this problem. The solution set is empty. This represents the case of 3 parallel planes and that can be seen from the structure of the equations.

**Example #2:** Let’s eliminate z.

A + C → 3x + 2y = 13 -9x – 6y = -39 So 0 = -12 which is also never true. But these 3

4A + B → 9x + 6y = 27 9x + 6y = 27 equations are not parallel, so they must form a “tent” , where all three planes never have any points in common. The answer is also “no solution” or the solution set is empty.

**Example #3:** This is similar to problem #2 in structure:

A + C → 3x + 2y = 9 -9x – 6y = -27 So 0 = 0 which is ALWAYS true. So these 3 planes

4A + B → 9x + 6y = 27 9x + 6y = 27 must all intersect in a line.

\*\*\*\*\*We can still write this solution!! If we know that 3x + 2y = 9, then we can let x be our independent variable and solve for y:  . Furthermore, we also know that x + y + z = 5, so . So the solution to #3 can be written as all ordered triples of the form:  . (so if x = 1, (1, 3, 1) lies on all 3 planes. We could also let y be the variable and write the ordered triple in terms of y. We would get . (If y = 3, (1, 3, 1) still lies on all 3 planes.)

**Example #4** Let’s eliminate z still.

A – C → x = 3 so equation A is now 6 + y – z = 4 or y – z = -2. Furthermore, B is now 3 + 2y + 4z = 11 or 2y + 4z = 8 → y + 2z = 4 and C is y – z = -2. Adding 2A + B → 3y = 0 or y = 0. By substitution, we get z = 2. Therefore this intersection is the point (3, 0, 2).